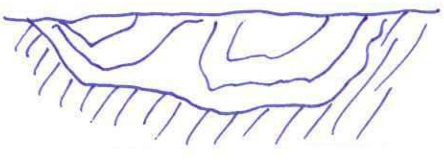
Stream discharge

Today we focus upon stream discharge — the $0.036 \cdot 10^6$ km³/yr $= 36{,}000$ km³/yr of runoff into the oceans

Note the units:

$$\text{flux (discharge)} = \frac{\text{volume}}{\text{time}} = \text{area} \times \text{velocity}$$

For a given river



or

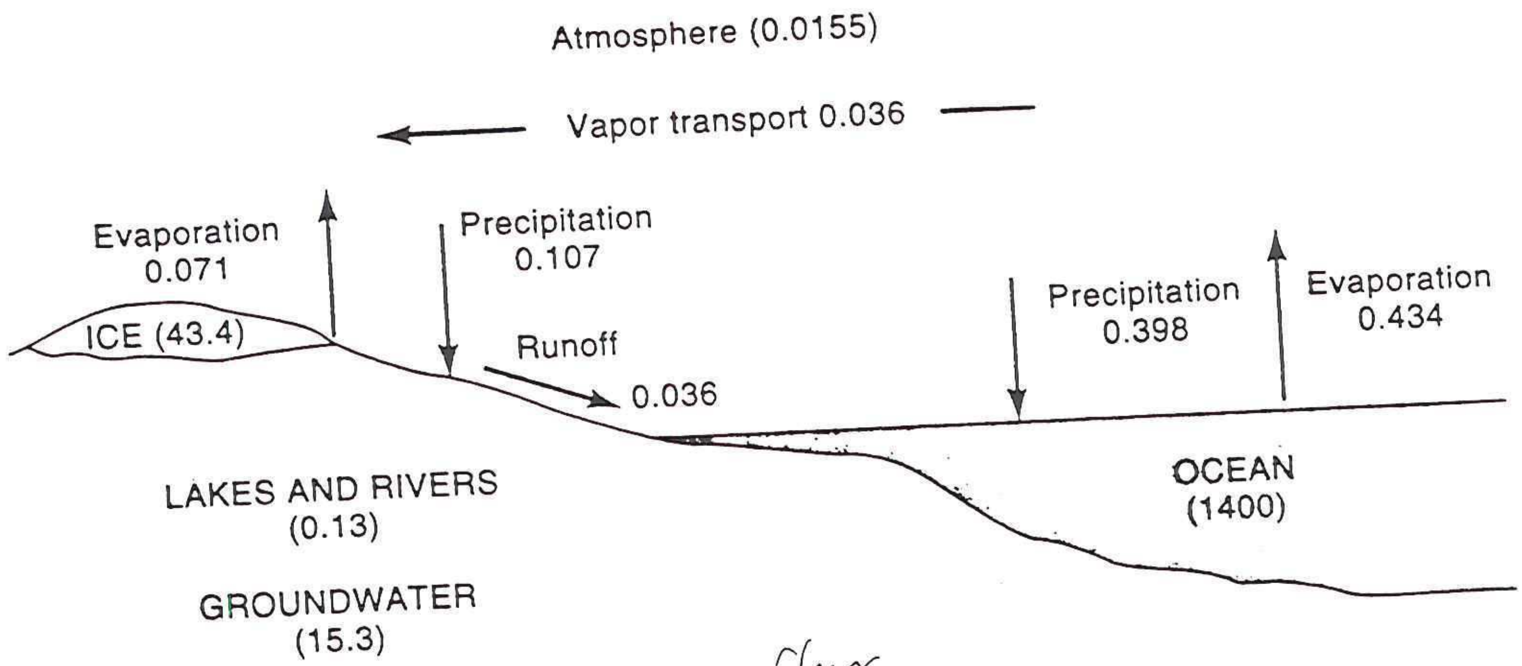
Discharge $Q = \sum v \, dA$

$$= \bar{v} \, A$$

$\bar{v}$: mean velocity; A: cross sectional area

The flux at a given locality, e.g., Stony Brook at Highway 206 is clearly variable in time. Example: Seneca Creek in Maryland — very similar stream

reservoir units : 10^6 km^3
flux units : 10^6 km^3/yr



river flux
36,000 km^3/yr

width & depth ≈ 30 x Stony Brook

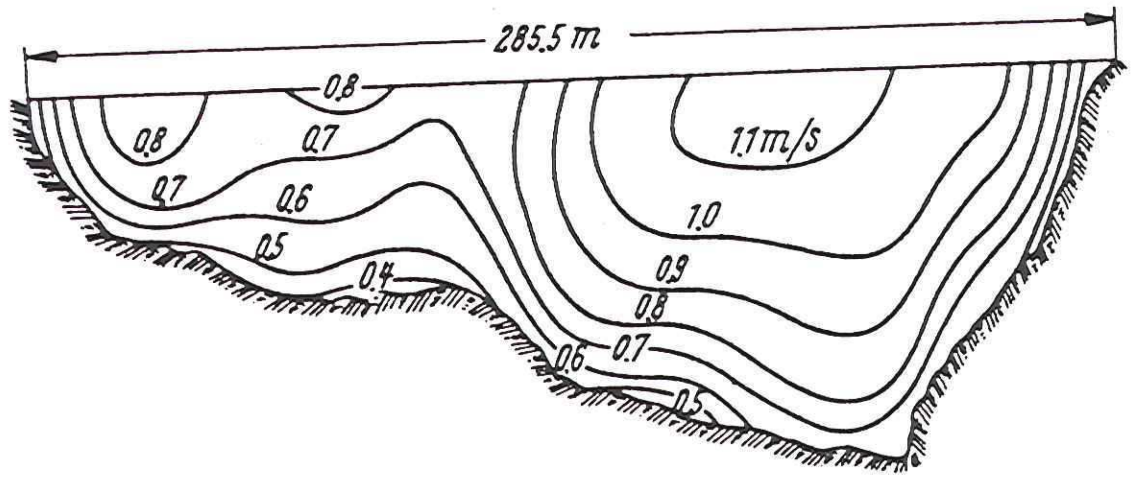


Fig. 51. Velocity distribution in a river. (After Schmidt 1957)

variability in time

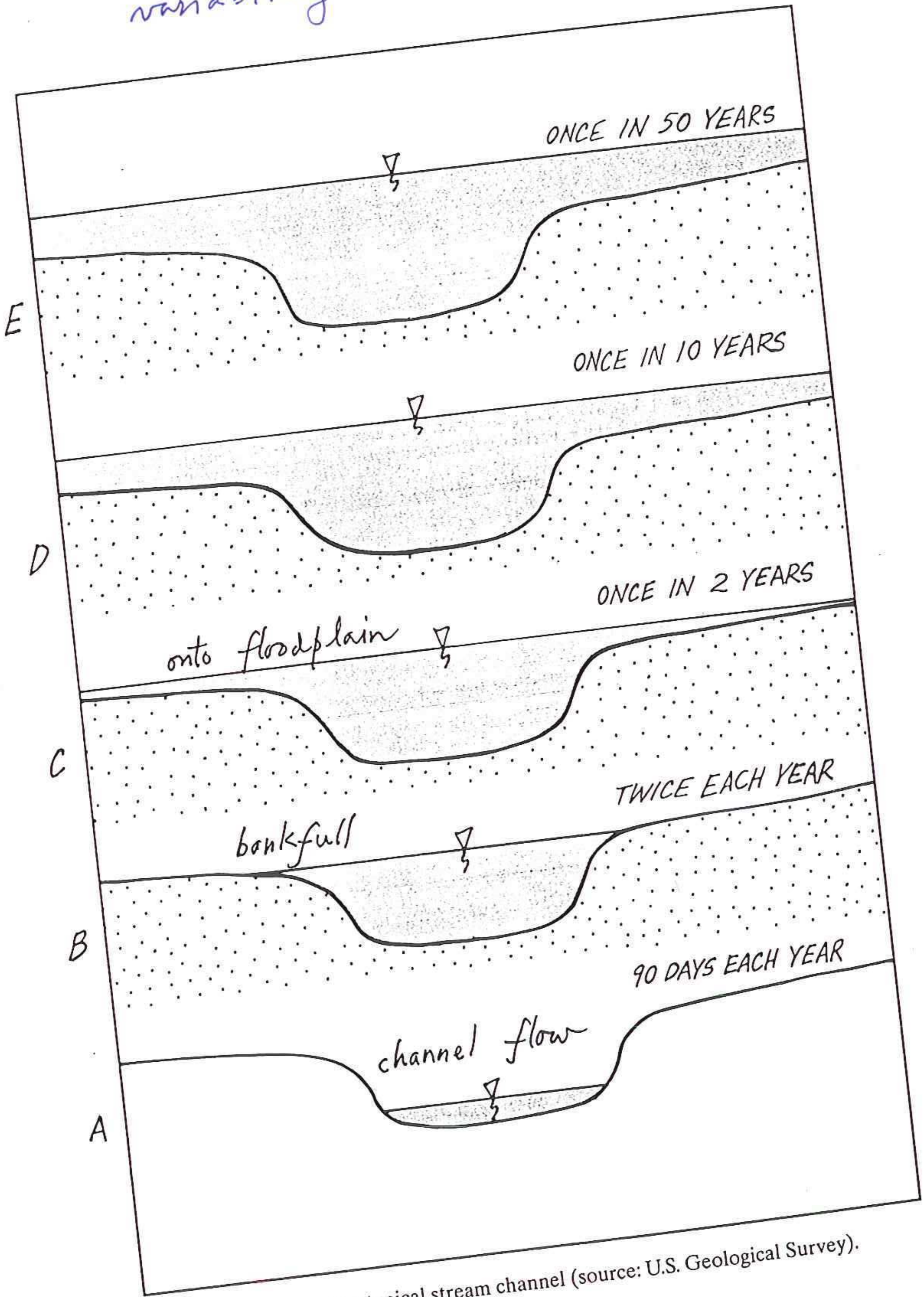
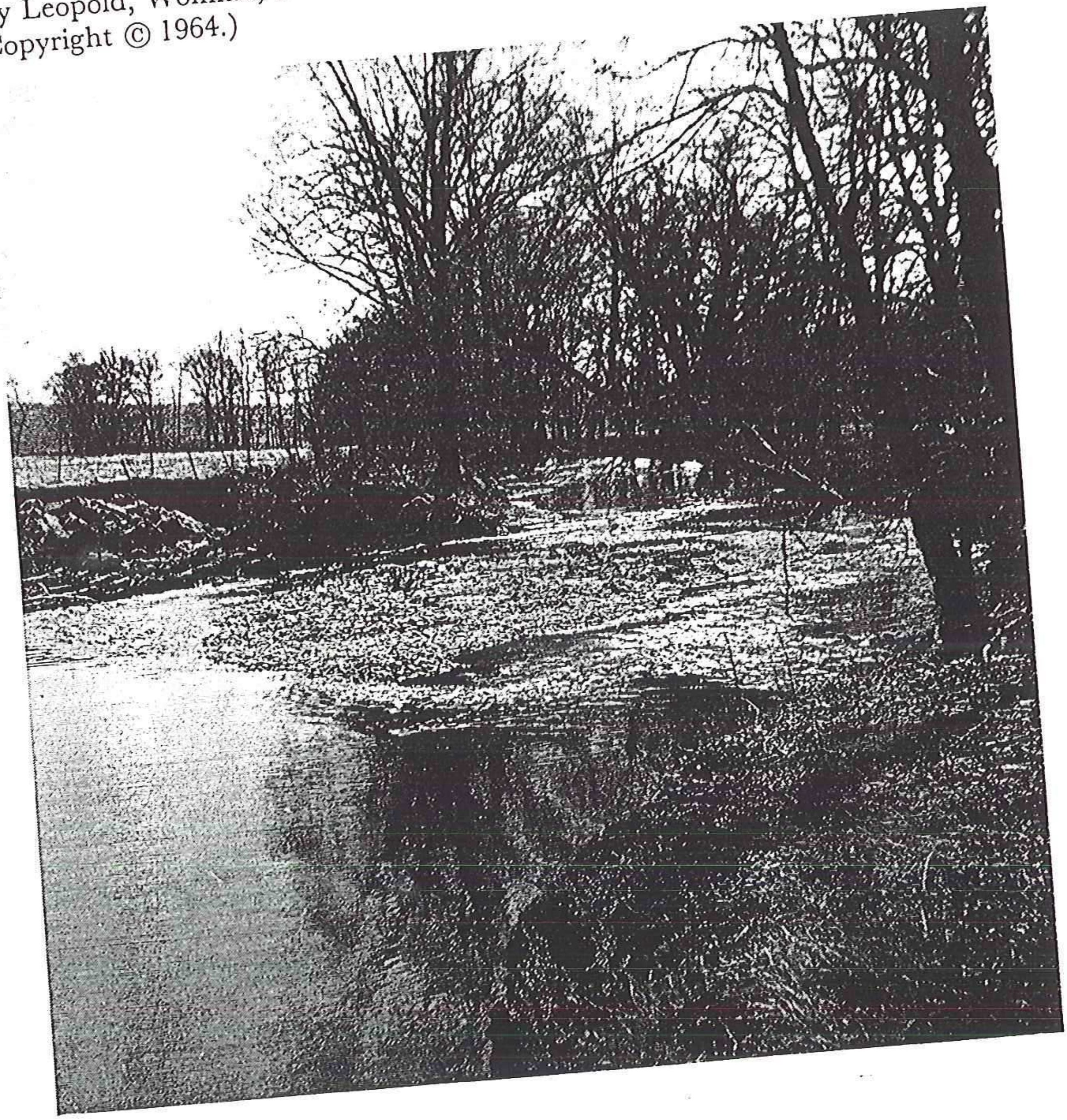


Figure 8.27 Levels of flow in a typical stream channel (source: U.S. Geological Survey).

Figure 11
Seneca Creek at Dawsonville, Maryland. View is downstream at low flow; bar in middle is composed of gravel. (From *Fluvial processes in geomorphology*, by Leopold, Wolman, and Miller. W. H. Freeman and Company. Copyright © 1964.)



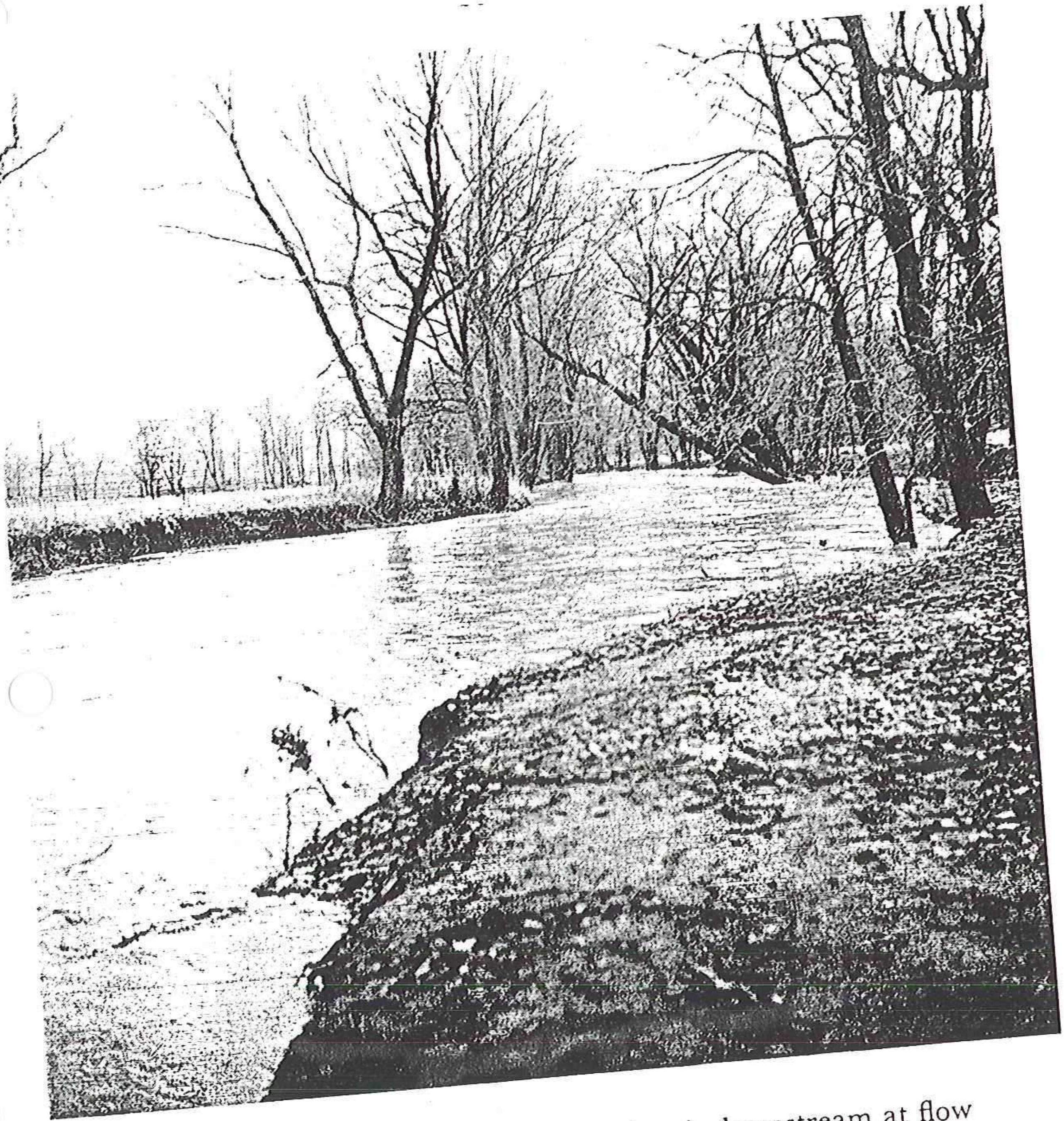


Figure 12
Seneca Creek at Dawsonville, Maryland. View is downstream at flow approximately half bankfull; note that riffle is drowned out so that no visible evidence of it appears on the water surface. (From *Fluvial processes in geomorphology*, by Leopold, Wolman, and Miller. W. H. Freeman and Company. Copyright © 1964.)

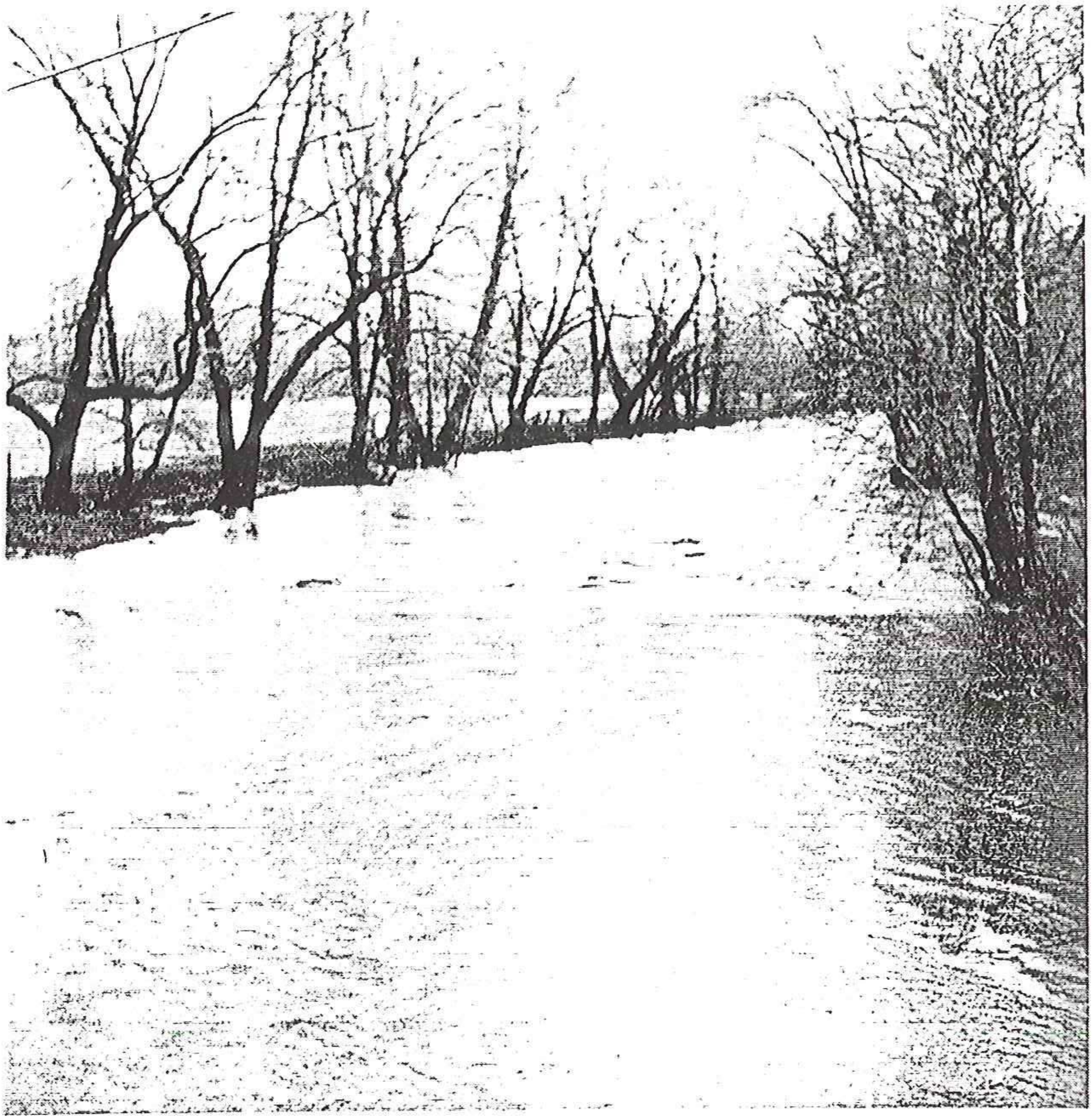


Figure 13
Seneca Creek at Dawsonville, Maryland. View is downstream at a stage near bankfull.

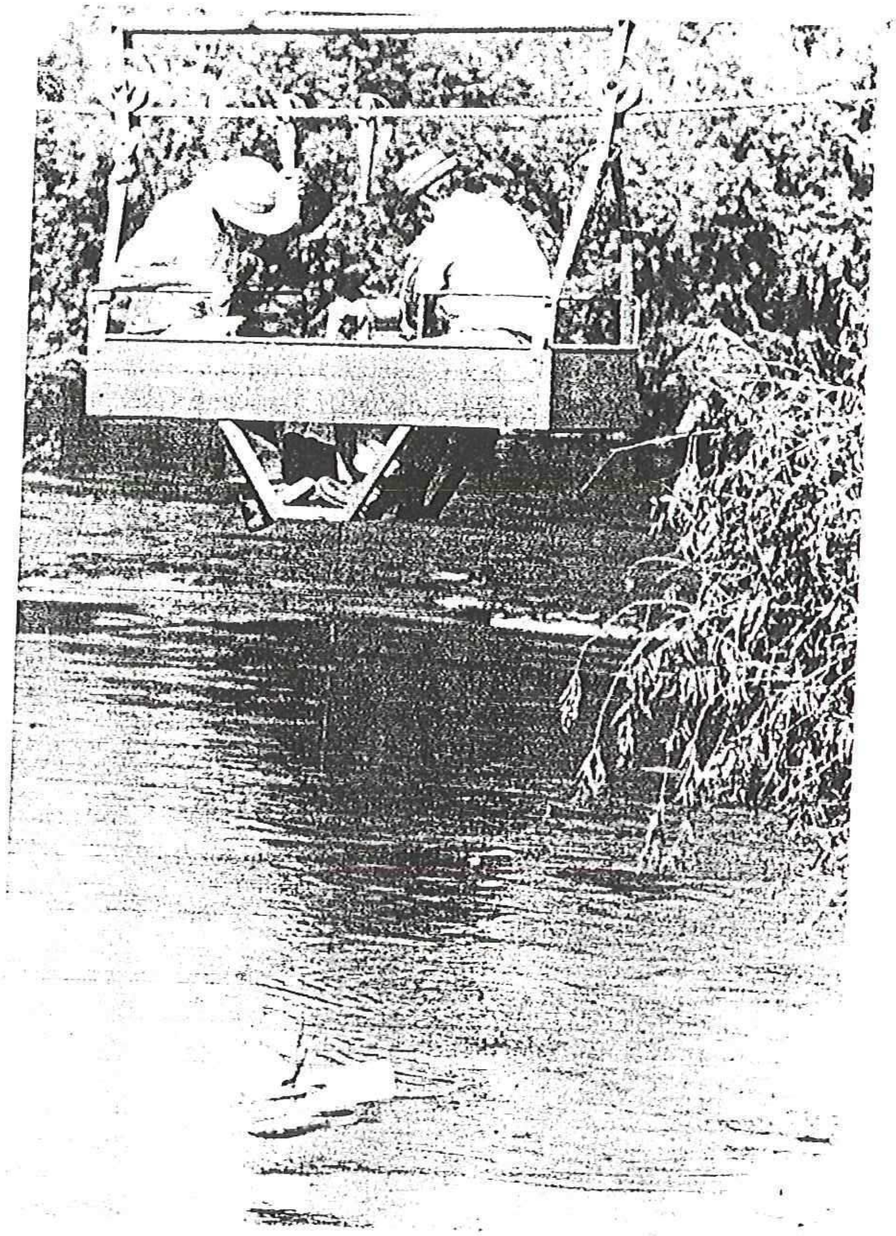


Figure 8.11 Lowering current meter in the river.

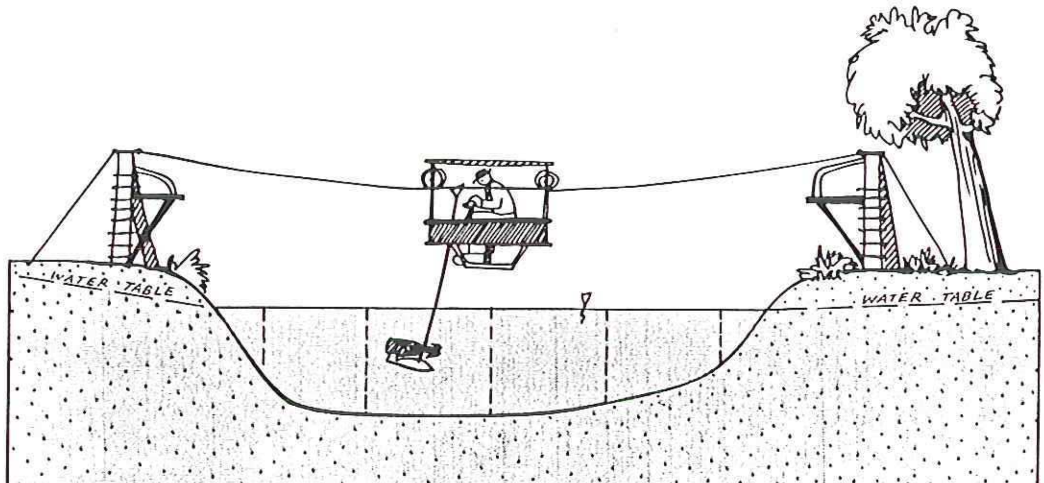


Figure 8.15 Sketch of stream cross-section for measuring stream discharge.

Many practical reasons for wishing to know
- predict availability of water supply
- design of bridges, dams, flood control (levees, etc.)
- have rare catastrophic events (100 year flood, 500 year flood) want statistics to characterize likelihood of these & plan for them

Suppose one truly wants to measure

$$Q = \sum v \, dA$$

Then must use a flow-meter to measure v at every location in stream cross-section.

Very labor-intensive measurement. What one really needs to compile statistics are are long time series $Q(t)$.

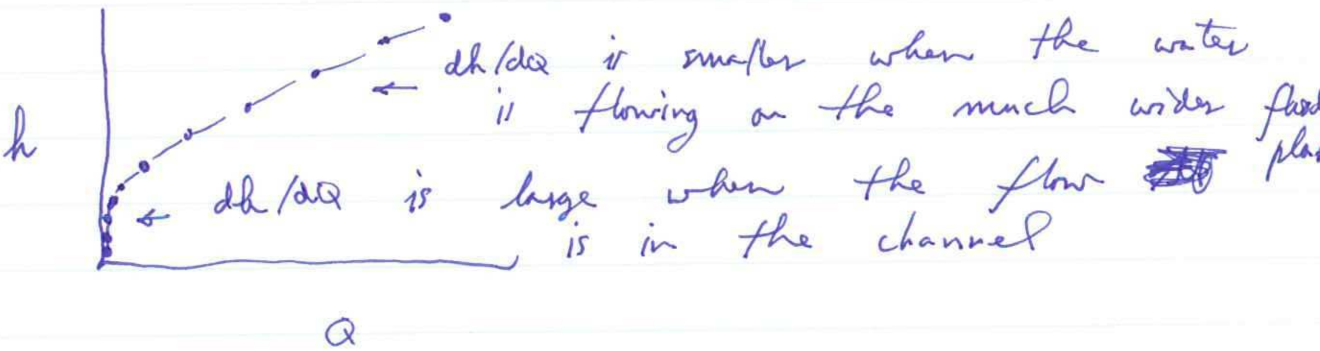
Cannot have men in hats measuring constantly. Instead one establishes an empirical relation between Q and a proxy, water height h. Then measure $h(t)$ and use the $Q-h$ relation to determine $Q(t)$.

Figure shows Q vs. h for Stony Brook

Gaging station (lab) measures h(t) — transmits data to geostationary satellite

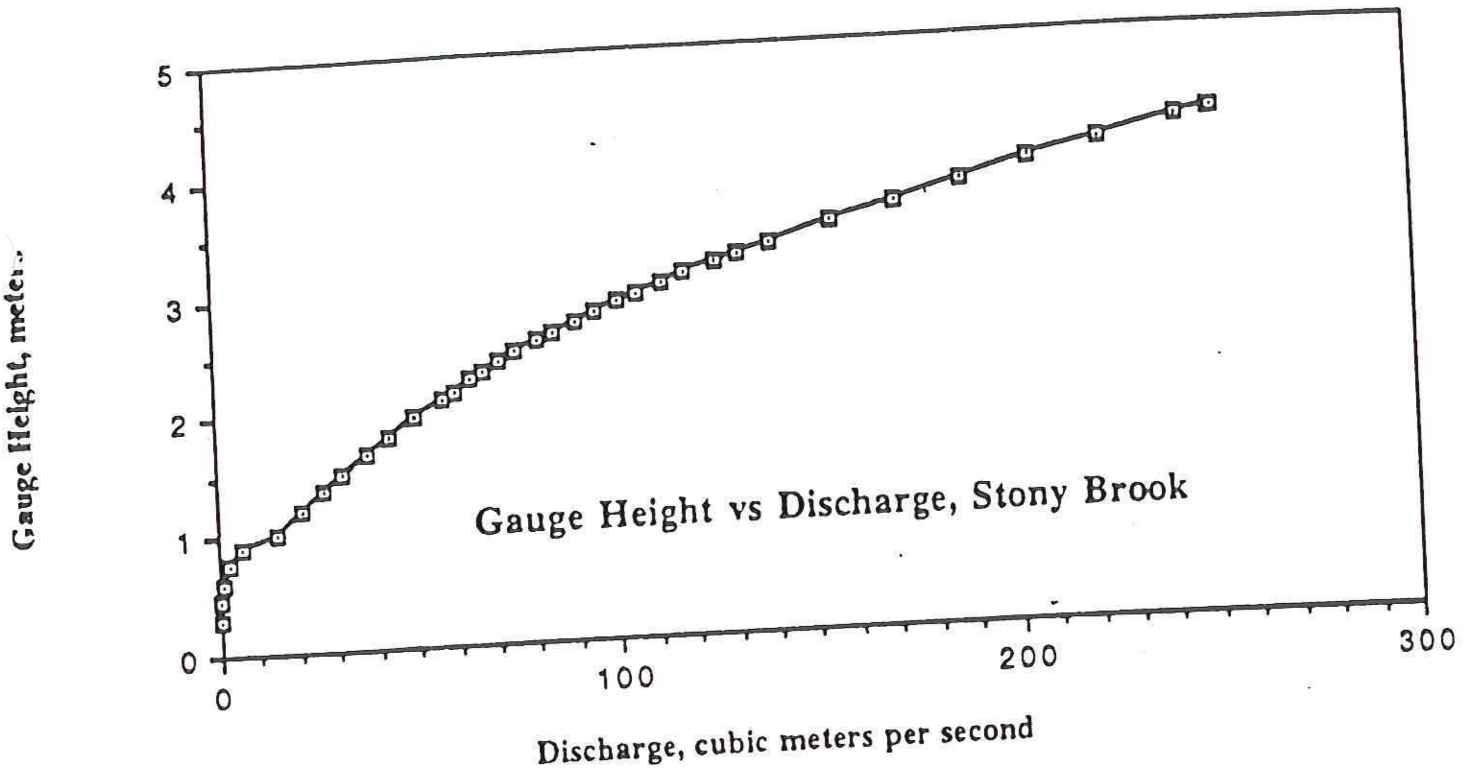
Two regimes:

- $h < 1$ meter, $Q < 20\ m^3/sec$
  flow confined to channel
- $h > 1$, $Q > 20\ m^3/sec$
  flow spills onto much wider floodplain



The change in character at $h = 1$ m, $Q = 20\ m^3/sec = 700\ ft^3/sec$ is seen more clearly in the linear log plot.

But note that this alters the look of the physically appealing slope change above



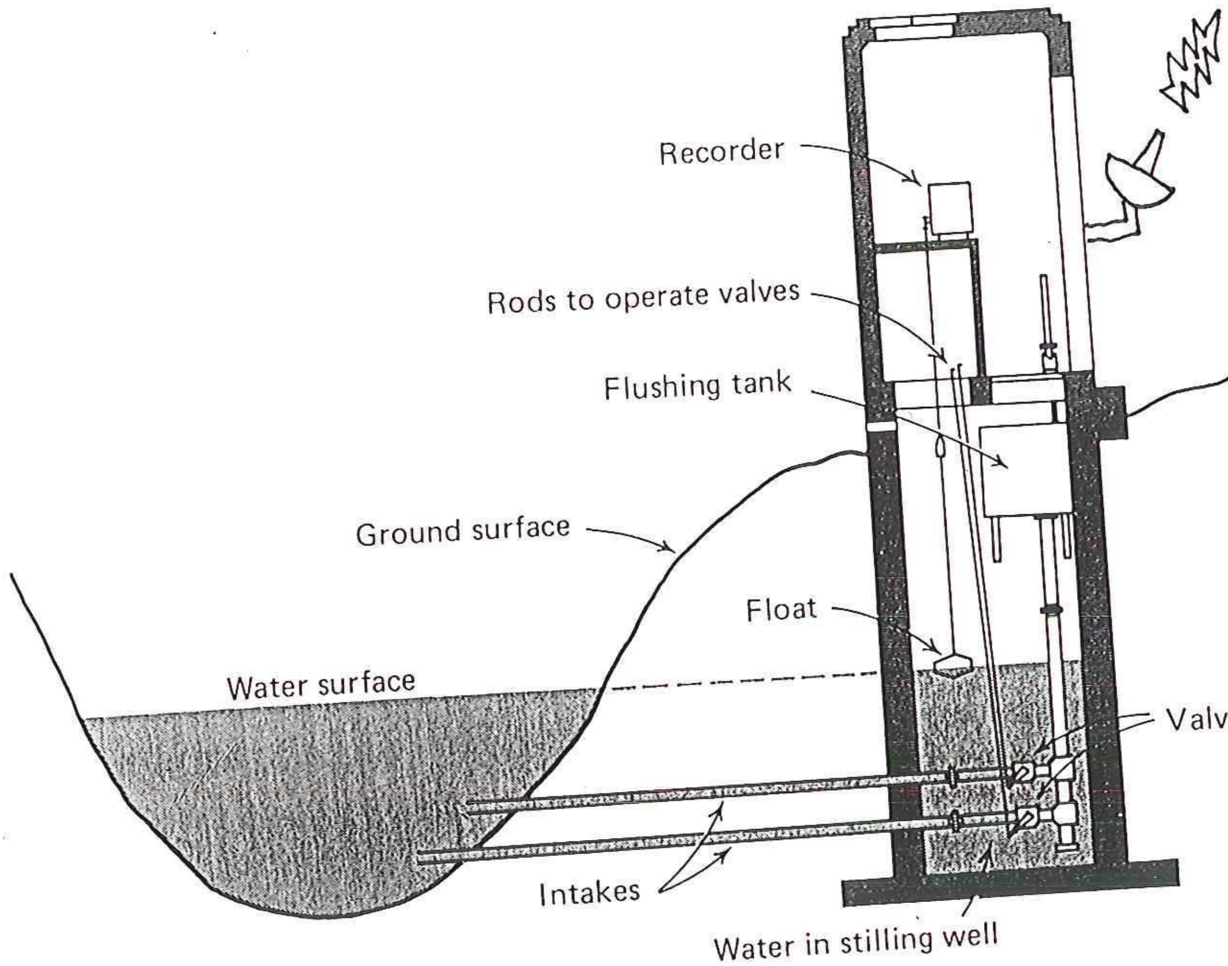
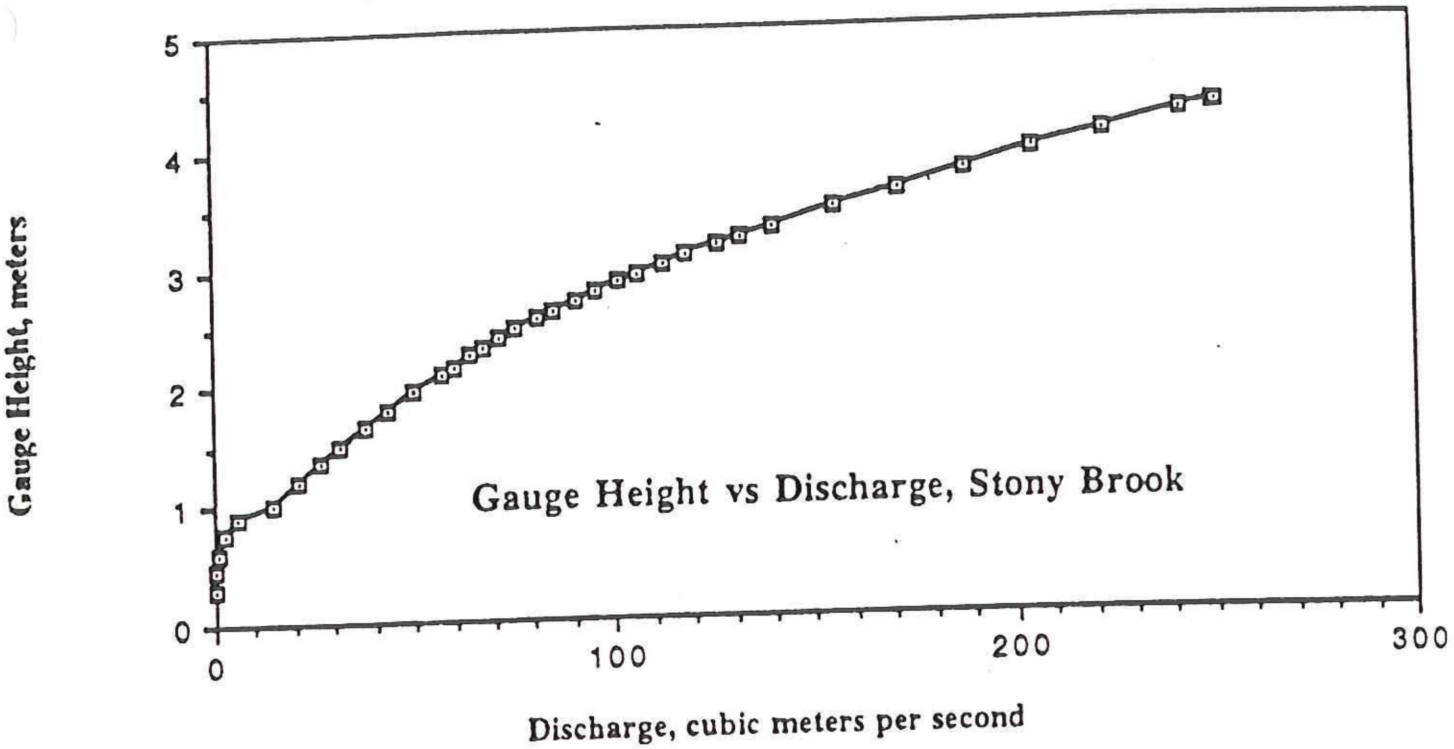
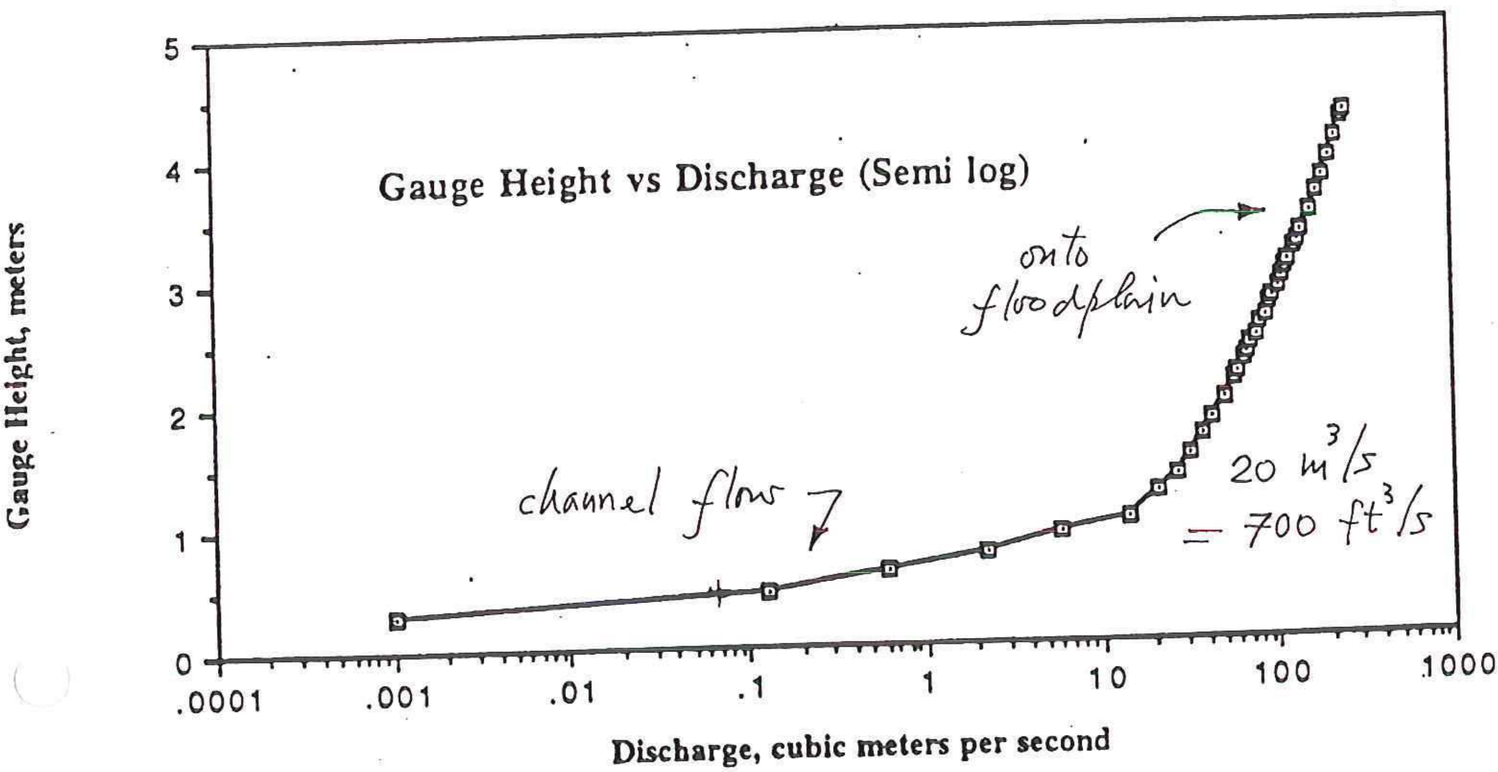


Figure 10
Diagram of a gaging station, showing relation of water in the stilling well to the river. (U.S.G.S.)


Gauge Height vs Discharge, Stony Brook
Gauge Height, meters
Discharge, cubic meters per second
0
1
2
3
4
5
0
100
200
300


Gauge Height vs Discharge (Semi log)
Gauge Height, meters
Discharge, cubic meters per second
0
1
2
3
4
5
.0001
.001
.01
.1
1
10
100
1000
onto floodplain
channel flow
20 m³/s
= 700 ft³/s

Is it possible to understand the shape of Q versus h curves such as that for Stony Brook ?

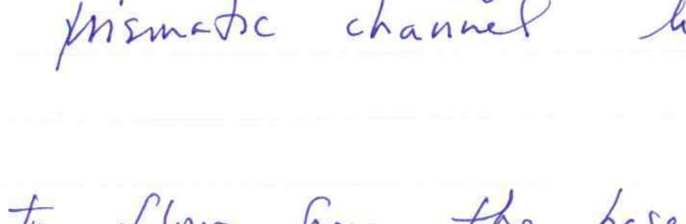
Many theoretical analyses, no firm understanding, very much an <u>empirical</u> engineering or applied science.

<u>Chézy (1769) analysis</u> : extremely simple – gives insight into the fundamental principles

Recall that the velocity contours in a simple prismatic channel look like Fig. 50

Resistance to flow from the base & channel walls — depends on roughness of channel (boulders, cobbles, sand, plants, etc.)

Consider a plug of water in the channel. Balance the driving & resisting forces.



laboratory scale model

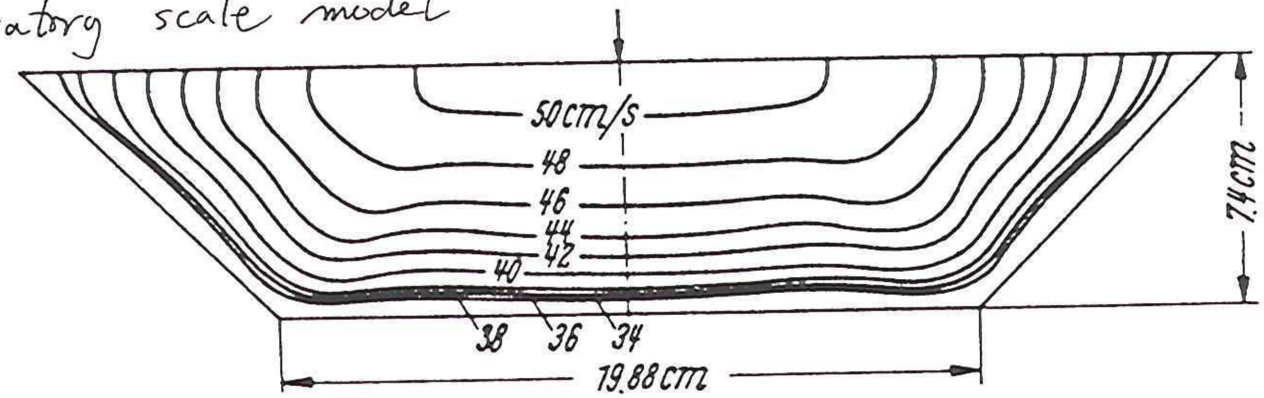
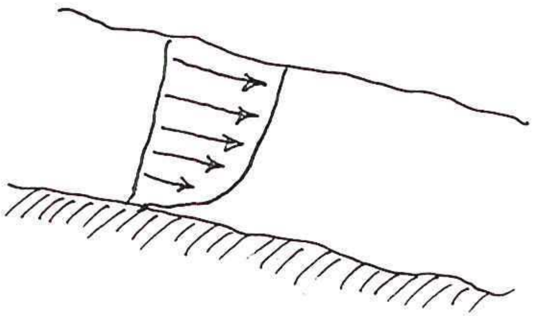
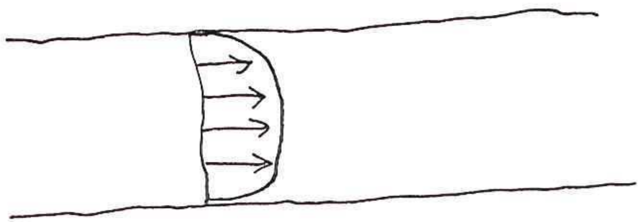


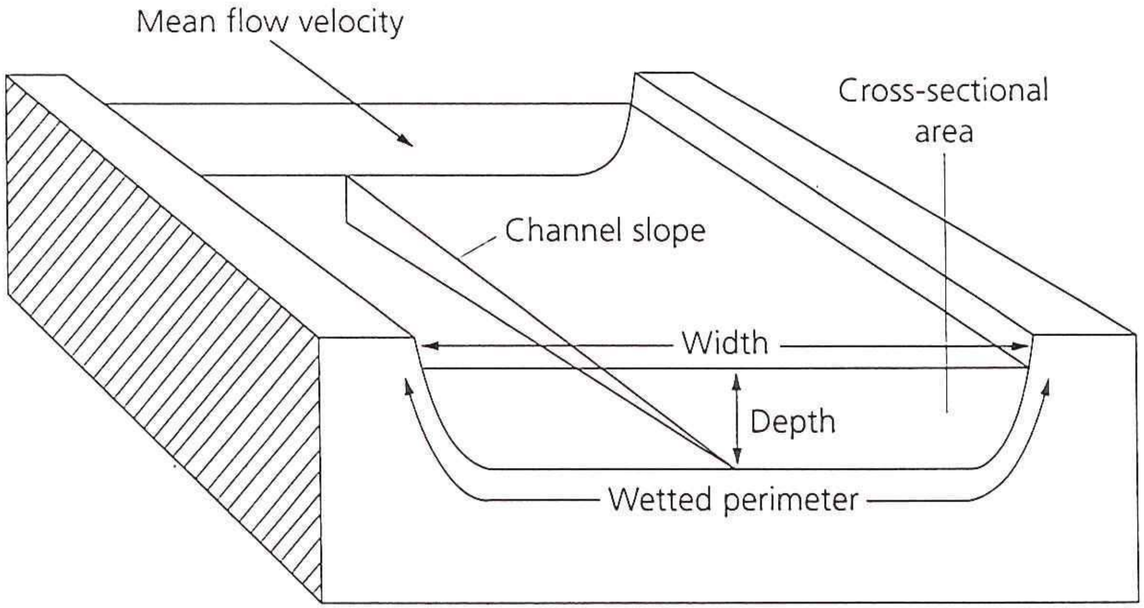
Fig. 50. Velocity distribution in a prismatic channel. (After Schmidt 1957)

Cross-section:



Map view:

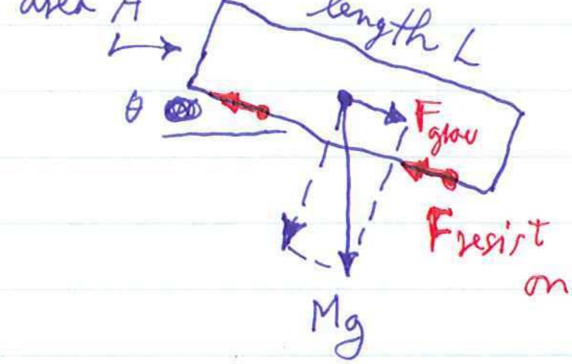




Discharge = Cross-sectional area × mean flow velocity
Hydraulic radius = Cross-sectional area/wetted perimeter

Fig. 5.8 The wetted perimeter and hydraulic radius in a river.

The driving force is gravitational



θ : slope of stream
M : mass of plug
ρ : density of H_2O

Driving force – in direction stream is flowing

$$F_{grav} = Mg \sin\theta = AL\rho g \sin\theta$$

The resisting traction (force/unit area) on the wetted perimeter is assumed to depend upon the flow rate:

true for a bicyclist or a 747 also

force/unit area

traction $\tau_{resist} = \frac{1}{2} C_{drag} \rho \bar{v}^2$

C_{drag} : the drag coefficient – dimensionless number of order 1

This a standard quasi-empirical relation for turbulent drag. The resisting force is

$$F_{resist} = \tau_{resist} PL = \frac{1}{2} C_{drag} PL \rho \bar{v}^2$$

↑ per unit area

Equating we find $AL\rho g \sin\theta = \frac{1}{2} C_{drag} P L \rho v^2$

$$v = \sqrt{\frac{2gA\sin\theta}{C_{drag} P}} = C\sqrt{\frac{A}{P}\sin\theta}$$

write in form $v = \sqrt{\frac{2g\sin\theta}{C_{drag}}}\sqrt{\frac{A}{P}}$ — $\sqrt{\frac{2g\sin\theta}{C_{drag}}}$: const at one site; $\sqrt{\frac{A}{P}}$: variable with time

where $C = \sqrt{\frac{2g}{C_{drag}}}$

full pipe or half-pipe: $R = \frac{A}{P} = \frac{\text{radius}}{2}$

half pipe

$A = \frac{1}{2}\pi R^2$

$P = \pi R$

$A = \frac{1}{2}\pi R^2$

$P = \pi R \Rightarrow \frac{A}{P} = \frac{R}{2}$

Conventional to introduce the so-called hydraulic radius

$$R = \frac{A}{P}$$

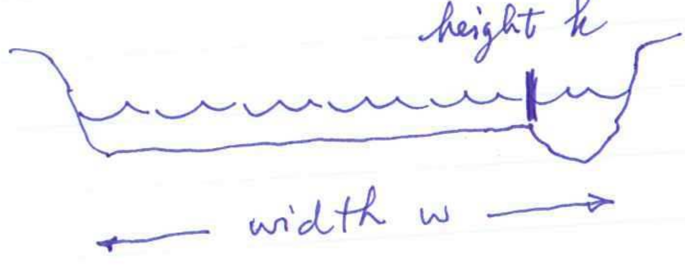
$$\bar{v} = \text{const}\sqrt{R\sin\theta}$$ — Chézy equation

discharge: $Q = vA = \sqrt{\frac{2g\sin\theta}{C_{drag}}}\sqrt{\frac{A^3}{P}}$ — $\sqrt{\frac{2g\sin\theta}{C_{drag}}}$: const; $\sqrt{\frac{A^3}{P}}$: variable

A simple semi-empirical relation between flow velocity, slope and channel geometry.

~~Model the Stony Brook cross-section by a sloping floodplain~~

Let us model the Stony Brook in flood stage as a broad flat floodplain — ignore the channel itself



area $A = wh$

perimeter $P = w + 2h \approx w$

hydraulic radius $R = \frac{A}{P} \approx h$

discharge $Q = vA$

$= \text{const} \sqrt{h \sin\theta} \cdot wh$

$Q = \text{const}\ h^{3/2}$ or

$Q = \text{const} \times w h^{3/2}$ ($w \approx$ const during spillover)

$$h = \text{const}\ Q^{2/3}$$

In fact, $h(m) = 0.12\ Q(m^3/sec)^{2/3}$

provides a pretty decent fit to the Stony Brook gauging curve.

The constant 0.12 depends on the stream gradient θ, and the drag coefficient C_{drag}, due to bushes, etc. on the floodplain

A more sophisticated strictly empirical formula, obtained by extensive studies in flumes

$$v = \text{const} \sqrt{R \sin\theta}$$ — Chézy

$$\bar{v} = \frac{1.486}{n} R^{1/6} \sqrt{R \sin\theta}$$

- $\bar{v}$ ↑ mean velocity
- n ↑ Manning roughness coefficient — see Table
- $R^{1/6}$ ↑ extra weak dependence on R
- $\sqrt{R \sin\theta}$ ↑ Chézy term

↑ Manning's empirical formula

$\bar{v}$ in ft/sec, R in ft — sorry!

n in $\sqrt[6]{\text{feet}}$!

Bigger $n \Rightarrow$ rougher — slows the flow down

Time series of Q(t) for 1993 Stony Brook — what do we see?

Highest flow in winter Dec – May
~ 100 ft^3/sec

Lowest in summer (dry season) ← an uncommonly dry summer
June – Sept 1–10 ft^3/sec

Remember — residence time of H_2O in atmosphere is 11 days

Each spike is a rainstorm, about 50/year

The discharge needed to top the bank and occupy the floodplain is 20 m^3/sec = 700 ft^3/sec. This only occurred 5 times in 1993. About every 10^{th} storm.

Lets analyze a typical flood stage more carefully. Fig 5.8 shows the Hurricane Agnes flood of June 1972 on Conestoga River near Lancaster PA

Several inches of rain fell over a 24–36 hour period. Note the ~24 hour time lag between the c.o.m. of the rainfall and the peak of the flooding (88,000 ft^3/sec ≈ ~~[illegible]~~ 10 × Stony Brook)

This is simply because it takes time for the H_2O, which falls all over catchment area, to get into the stream.

Note division of storm flow into base flow, interflow and overland runoff.

These concepts are clarified in Figure 5.7

Note that even during low flow stage, there is ground water flow into the stream.

Most of the flow in tributaries such as the Stony Brook comes from storm events.

Usually asymmetrical e.g., Stony Brook

sinking limb reflects continuing interflow & groundwater inputs after storm

Discharge of major rivers at their mouths (e.g., Mississippi @ New Orleans) reflects input from upstream tributaries. Fig. 14.19 shows the major drainage basins in US. Note — Missouri to Columbia — route of Lewis & Clark expedition ~ 1800.

The Hurricane Floyd flood is not asymmetrical why? intense very sudden storm — all overland flow

Table 1.4 lists discharges of major rivers, their drainage areas and lengths.

Amazon is biggest by far — 6900 km^3/yr

This is 20% of the total 36,000 km^3/yr

lab exercise next week

Next — Congo, Ganges, Yangtze, Orinoco, Parana ... Mississippi is 8th

Mississippi 12 times smaller than Amazon

Conclude with discussion of flood recurrence intervals, a practical application of stream gaging records $Q(t)$.

Example : Stony Brook.

actually, I think the post-Floyd transparencies use $R = \frac{N}{rank} = \frac{46}{rank}$

Given an N year record, find the largest flood in every year. Rank order these from largest to smallest and plot Q versus the recurrence interval

$$R = \frac{N+1}{M} = \frac{\#\ years + 1}{rank}$$

R is a crude measure of the average time between flood occurrences of magnitude or rank M

The amount of suspended sediment is greater near the base because of settling.

Not surprisingly, there are tremendous variations in the three loads with time, associated with storms.

Example – the 1941 flood on the San Juan River near Bluff, Utah

Figure A shows the variation in suspended load transport (tons of sediment per day) and discharge (cubic feet per second)

The suspended load and discharge both went up, then down

The maximum discharge was ~ 50,000 ft^3/sec

$$5 \cdot 10^4 \frac{ft^3}{sec} \times 62.4 \frac{lb}{ft^3} \times \frac{1}{2200} \frac{tons}{lb} \times 86400 \frac{secs}{day}$$

$$= 1.2 \cdot 10^8 \text{ tons of } H_2O/\text{day}$$

The suspended load then was 10^7 tons/day

The "water" was a slurry 8% [illegible] [illegible]. mud by weight

This plots nicely on the observed boundary.

This part of the curve where the erosion & deposition curves merge is the province of bedload transport.

The remainder of the diagram pertains to the suspended load.

think of mudballs, clods, versus non-cohesive mixture of sand.

It is observed empirically that fine silt is less susceptible to erosion than fine sand — this a consequence of its greater cohesion.

See page 7.5 for explanation

The boundary between suspended transport and deposition is dashed because it does not depend only on $\bar{v}$. The suspended particles are always settling out — boundary depends on water depth and how far you want to transport them.

At a bend in a river, a water particle of mass m experiences a centrifugal force

[2] Many reservoirs have been built in the last 75 years for flood control, water supply and power production. These reservoirs, like any lakes, are also effective sediment traps, which raises the question of how long it takes for them to fill with sediment. The answers are enormously variable because erosion rates vary over many orders of magnitude, worldwide. Consider the example of Hoover Dam which was completed on the Colorado River near Las Vegas in 1936 creating Lake Mead, largely for water and power for Los Angeles, but also for irrigation. The engineers who designed the dam had to determine the lifetime of the reservoir, using data something like these:

reservoir volume = 3.5×10^{10} m^3
mean flow of Colorado River = 2.1×10^{13} liters / year
mean sediment concentration = 13.9 grams / liter
density of sediment = 2000 kg / m^3

When will Lake Mead be completely filled with sediment?

[3] In the nineteenth century, before the discovery of radioactive decay, a variety of attempts were made to constrain the magnitude of geologic time. For example the rates of sediment and salt supply by rivers to the oceans were used to calculate the times required to produce (i) all the known sedimentary rocks and (ii) all the salt in the oceans plus salt deposits (which are derived from evaporation of sea water). The results of these calculations were in effect (i) the mean "residence time" of salt in the oceans and (ii) the mean "life expectancy" for sedimentary rocks, because both salt and rock are recycled many times over the age of the Earth. How many times would sediment be recycled over the age of the Earth, assuming that present conditions held for the entire time?

total existing mass of sediments and sedimentary rocks = 2.4×10^{24} grams
world mean sediment concentration in rivers = 0.40 grams per liter
world river flux = 3.6×10^{16} liters per year

By means of comparison, what is the mean residence time of water in the oceans and how many times would water be recycled over the age of the Earth, assuming present conditions? The volume of the world's oceans is about 1.4×10^9 km^3. Why is water recycled faster than sedimentary rocks?

[4] Considering that all sedimentary rocks ultimately must be derived by weathering from igneous rocks, then the mass of ignous rock weathered should equal the sum of the masses of the derived sediments:

$$M_{\text{igneous rock}} = M_{\text{shale}} + M_{\text{sandstone}} + M_{\text{other}}$$

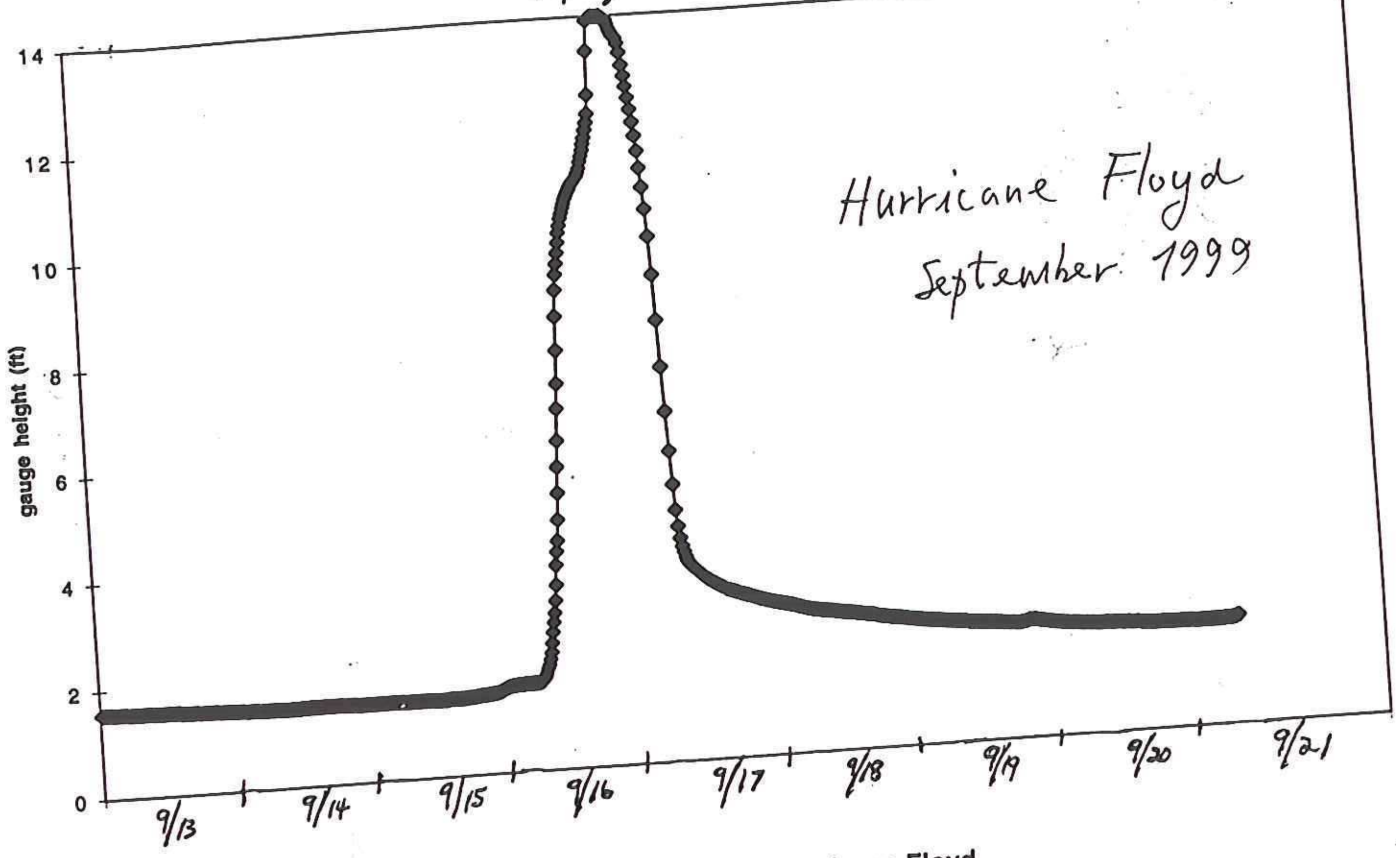
Dividing by the $M_{\text{igneous rock}}$ we get the mass fractions X of the sediment types:

$$1 = X_{\text{shale}} + X_{\text{sandstone}} + X_{\text{other}}$$

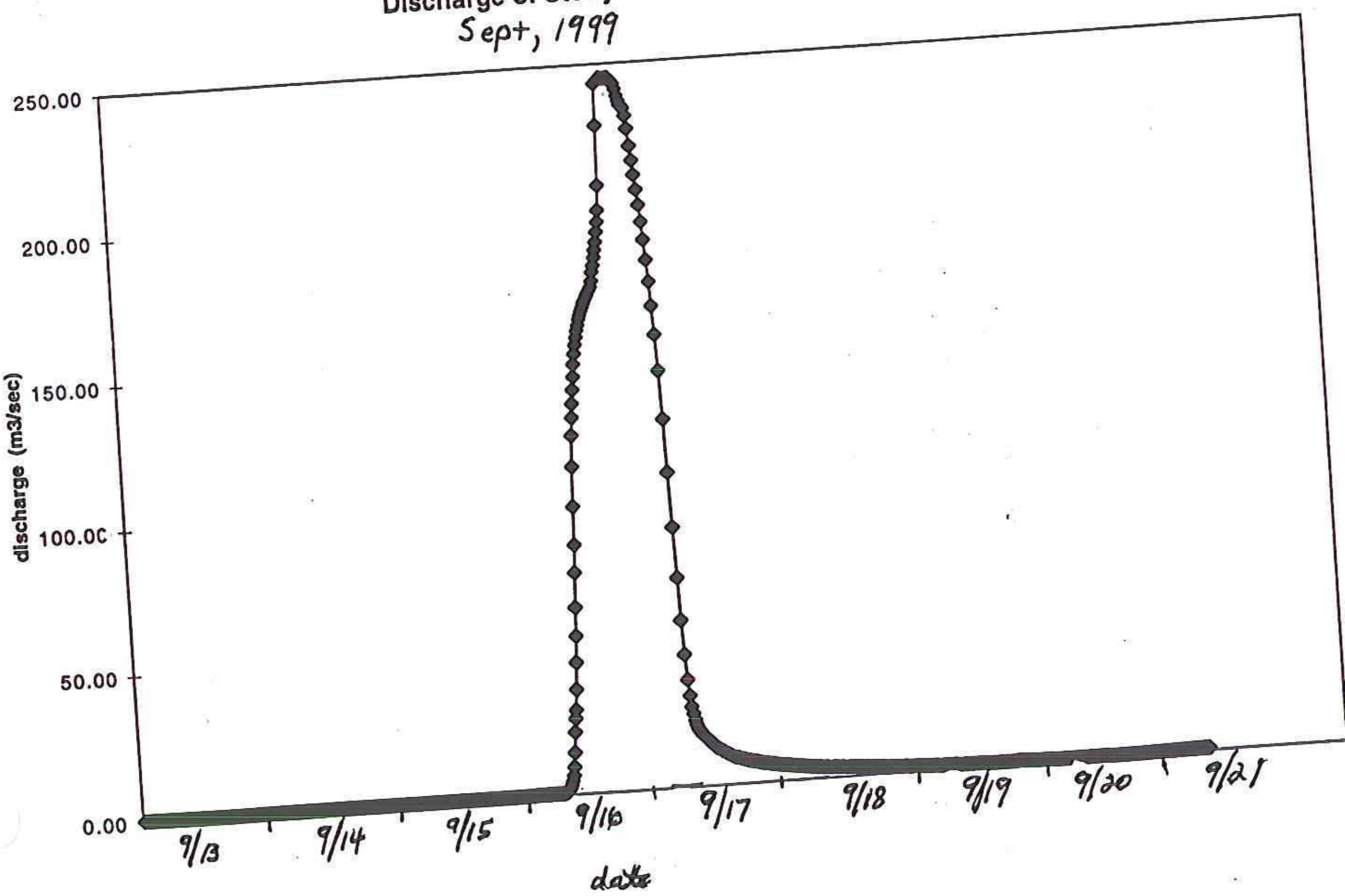
Also, for each chemical species, say SiO_2, the mass fraction of the species is just the percent in the rock times the mass, so:

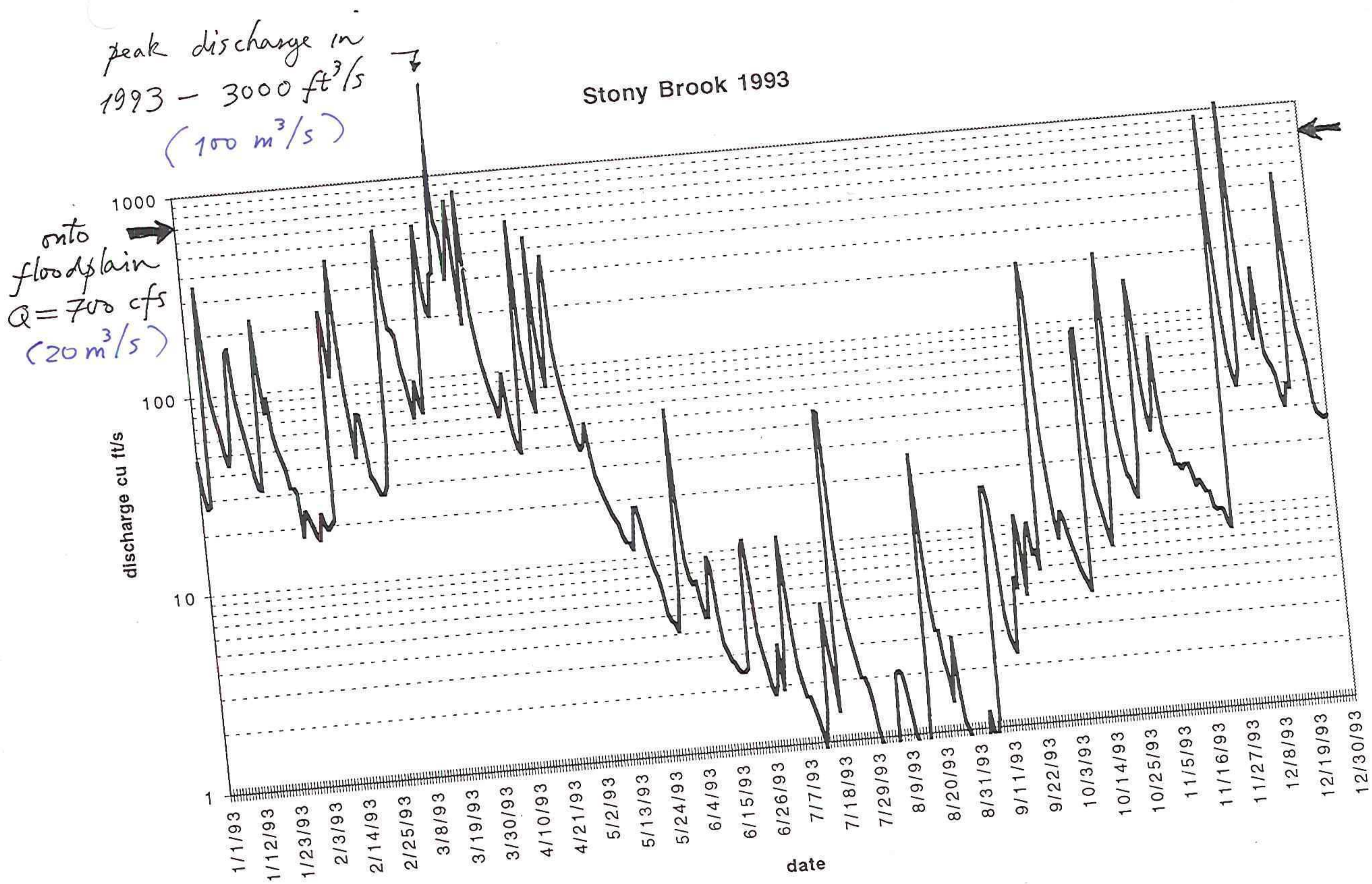
$$1*\%(SiO_2)_{\text{igneous rock}} = X_{\text{shale}}*\%(SiO_2)_{\text{shale}} + X_{\text{ss}}*\%(SiO_2)_{\text{sandstone}} + X_{\text{other}}*\%(SiO_2)_{\text{other}}$$

Gauge Height on Stony Brook, Hurricane Floyd
Sept, 1999



Discharge of Stony Brook, Hurricane Floyd
Sept, 1999





peak discharge in 1993 – 3000 ft³/s (100 m³/s)
Stony Brook 1993
onto floodplain Q = 700 cfs (20 m³/s)
1000
100
10
1
discharge cu ft/s
1/1/93
1/12/93
1/23/93
2/3/93
2/14/93
2/25/93
3/8/93
3/19/93
3/30/93
4/10/93
4/21/93
5/2/93
5/13/93
5/24/93
6/4/93
6/15/93
6/26/93
7/7/93
7/18/93
7/29/93
8/9/93
8/20/93
8/31/93
9/11/93
9/22/93
10/3/93
10/14/93
10/25/93
11/5/93
11/16/93
11/27/93
12/8/93
12/19/93
12/30/93
date

Table 1.4 Mean annual runoff of the world's largest rivers. From Shiklomanov (1993) [21].

River	Average runoff ($km^3 y^{-1}$)	Area of basin ($10^3 km^2$)	Length (km)	Continent
Amazon	6930	6915	6280	South America
Congo	1460	3820	4370	Africa
Ganges (with Brahmaputra)	1400	1730	3000	Asia
Yangzijiang	995	1800	5520	Asia
Orinoco	914	1000	2740	South America
Paraná	725	2970	4700	South America
Yenisei	610	2580	3490	Asia
Mississippi	580	3220	5985	North America
Lena	532	2490	4400	Asia
Mekong	510	810	4500	Asia
Irrawaddy	486	410	2300	Asia
St Lawrence	439	1290	3060	North America
Ob	395	2990	3650	Asia
Chutsyan	363	437	2130	Asia
Amur	355	1855	2820	Asia
Mackenzie	350	1800	4240	North America
Niger	320	2090	4160	Africa
Columbia	267	669	1950	North America
Magdalena	260	260	1530	South America
Volga	254	1360	3350	Europe
Indus	220	960	3180	Asia
Danube	214	817	2860	Europe
Salween	211	325	2820	Asia
Yukon	207	852	3000	North America
Nile	202	2870	6670	Africa

FIGURE 14.43 In the Americas most of the drainage from the continents is directed to the Atlantic and Arctic oceans, and very little to the Pacific Ocean.

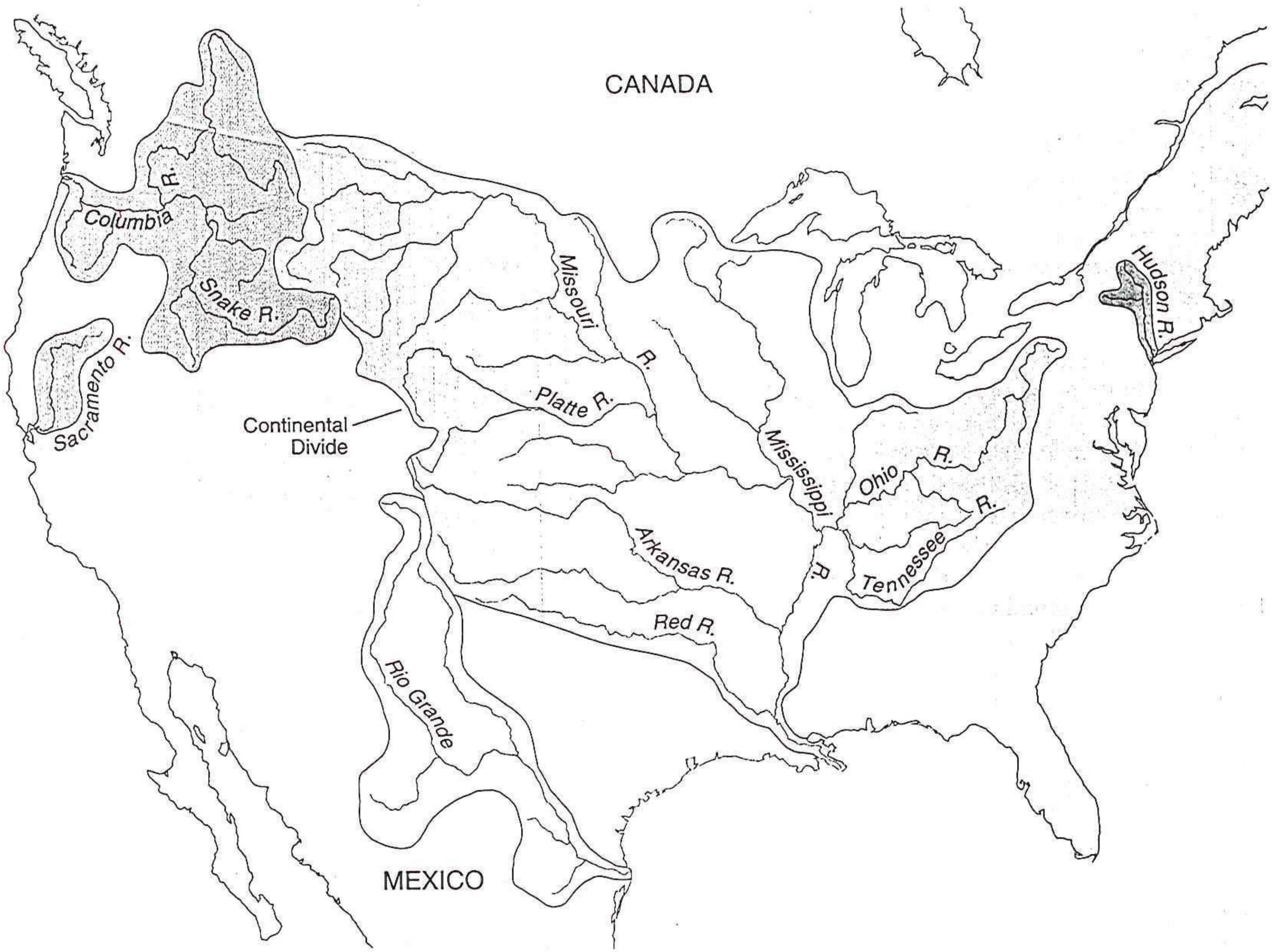
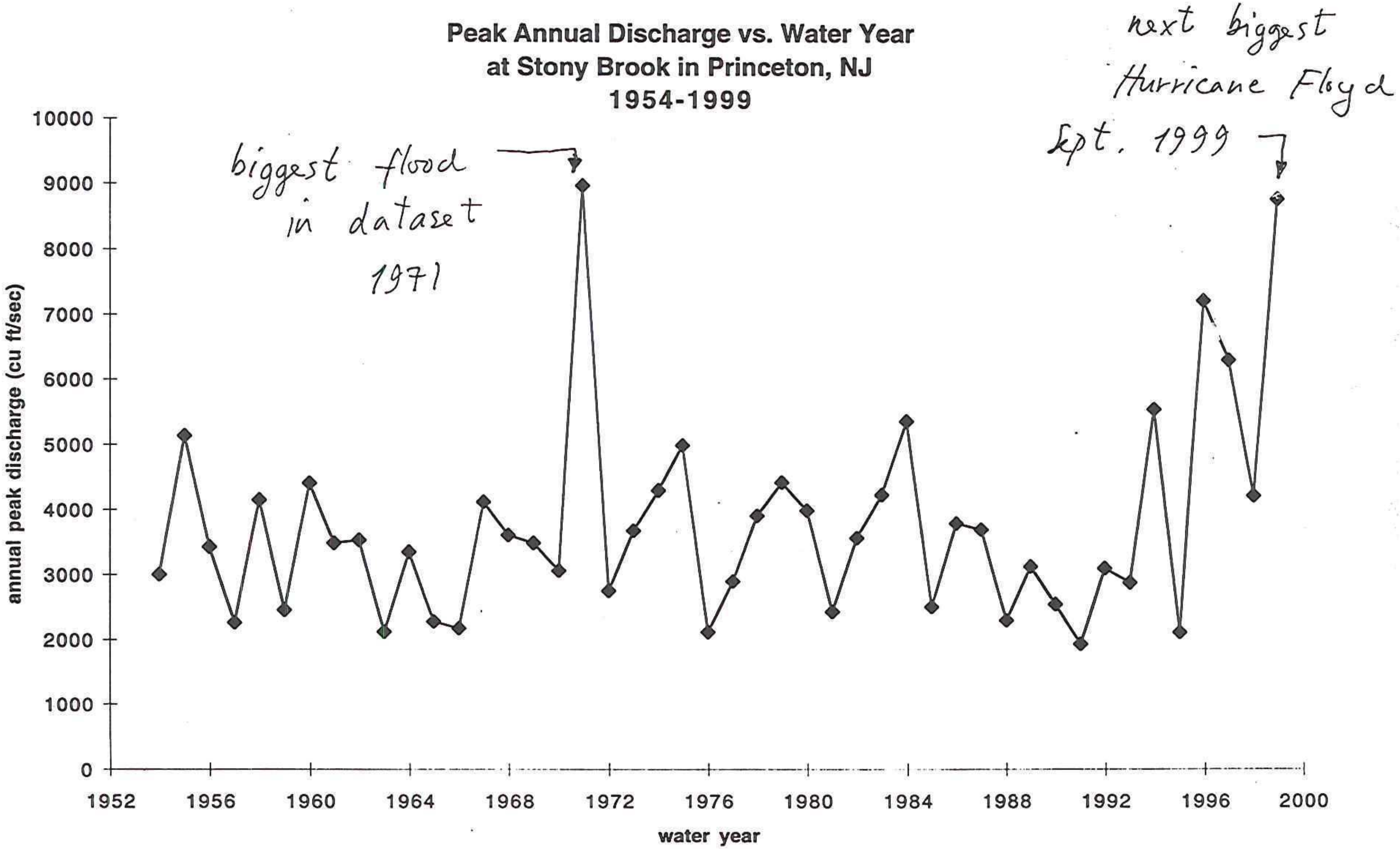
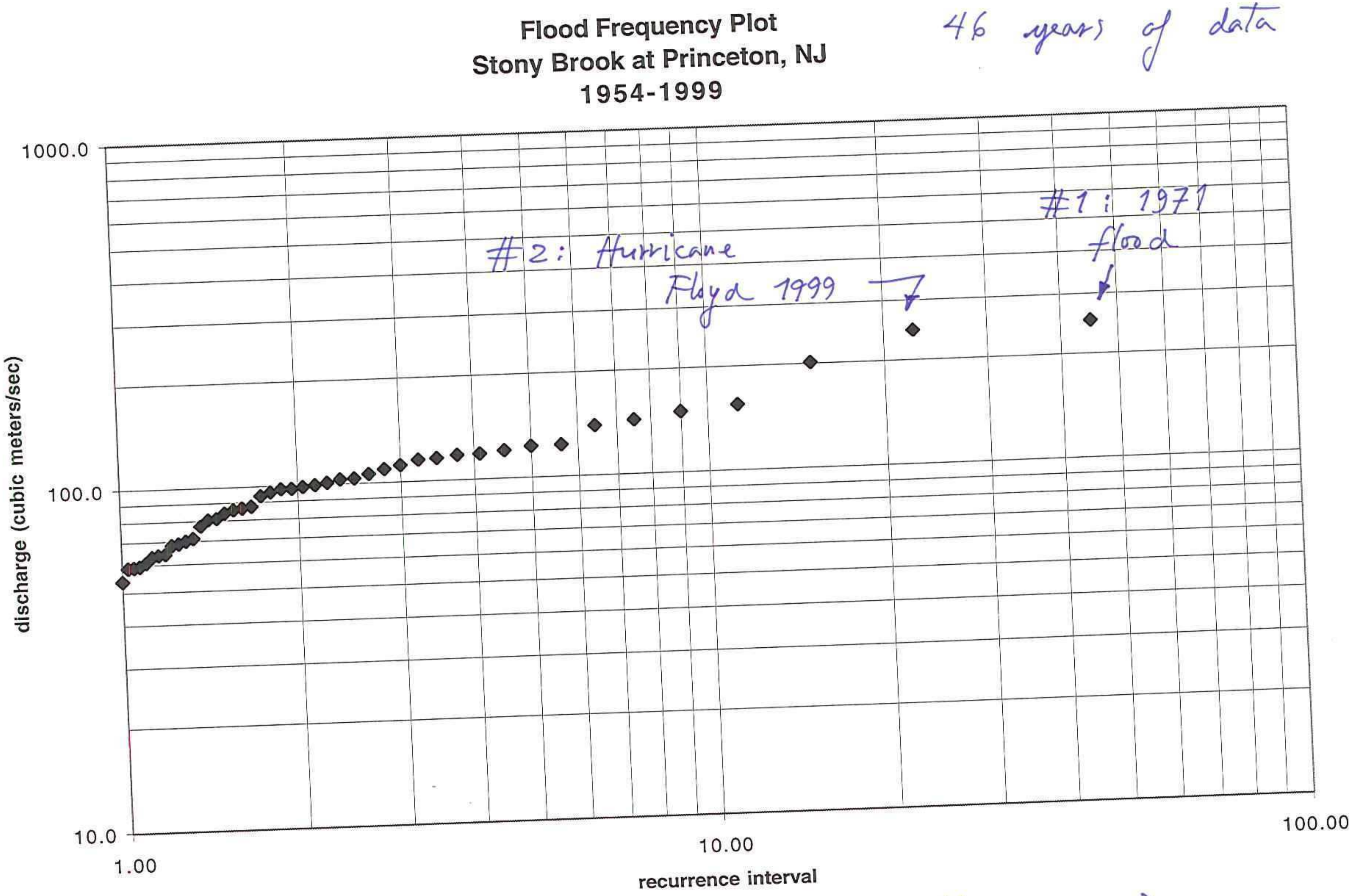


FIGURE 14.19 The stream's drainage basin is the area from which the stream and its tributaries receive water. Some well-known drainage basins in North America are shown here.



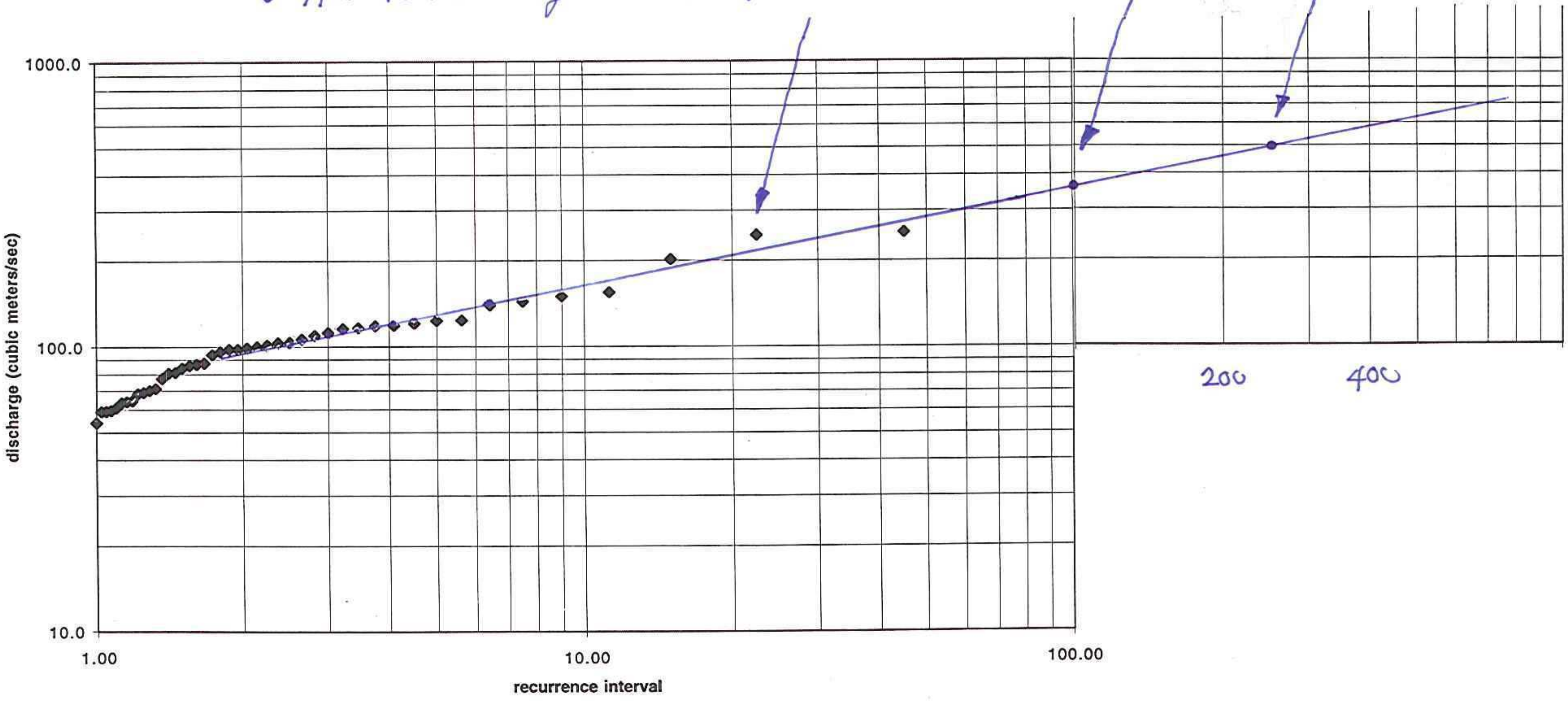
1954–1999 – 46 years of data



flood recurrence interval = $\frac{46}{\text{rank}}$ (years)

Flood forecasting – by extrapolation

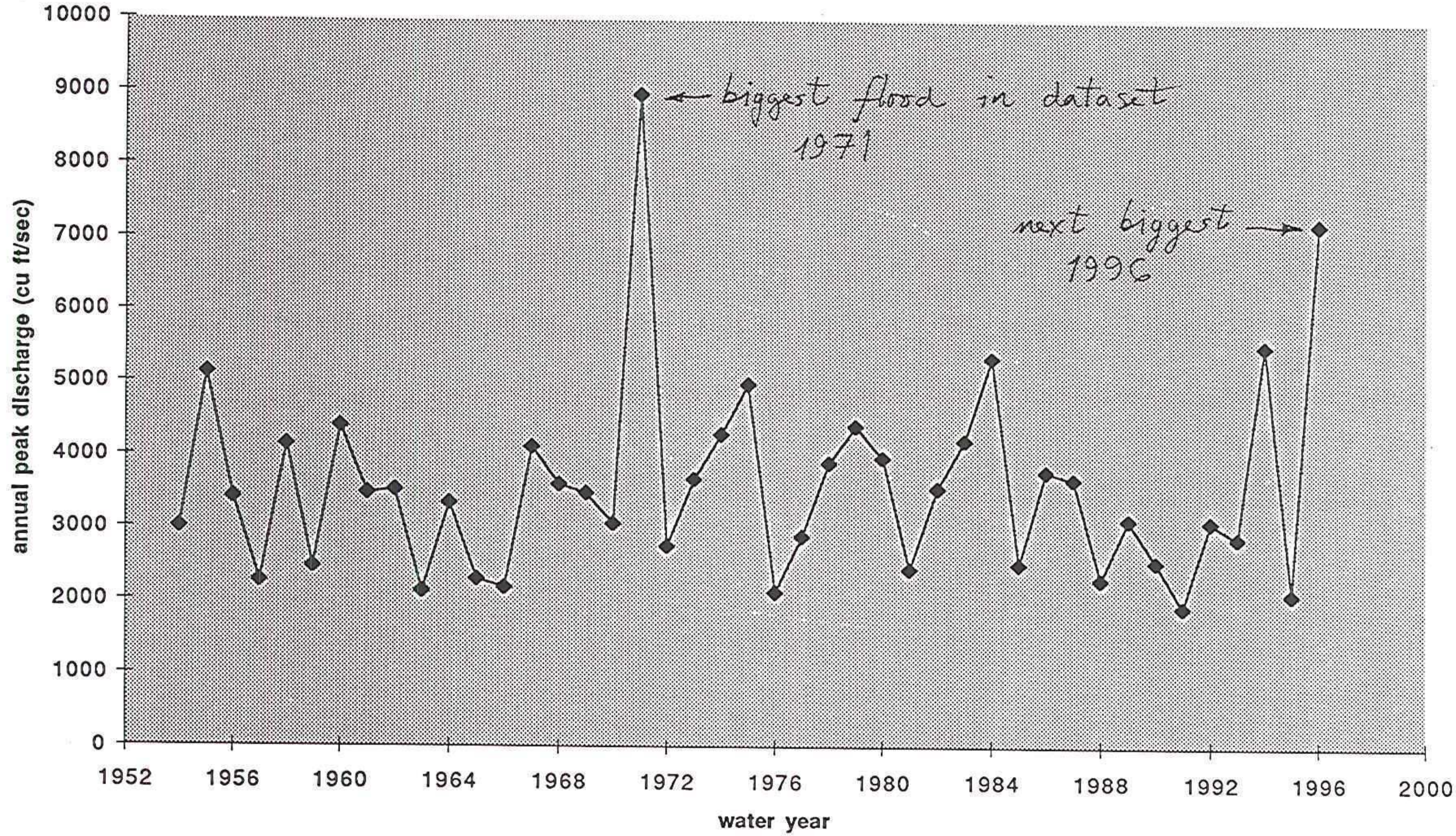
- flood twice as big as largest ever recorded (i.e., 500 m^3/sec) about every 250 years
- 100-year flood (about 360 m^3/sec)
- Hurricane Floyd 1999 (250 m^3/sec)



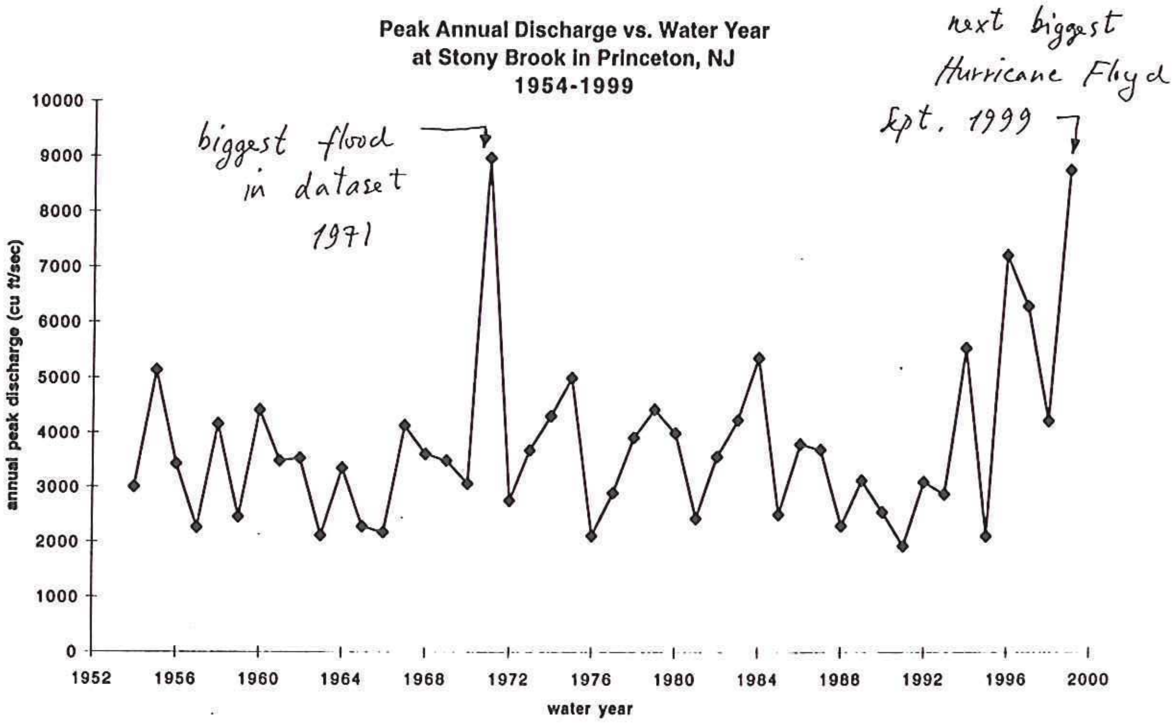
flood recurrence interval = $\frac{46}{rank}$ (years)

Peak Annual Discharge vs. Water Year at Stony Brook in Princeton, NJ

pre-Floyd



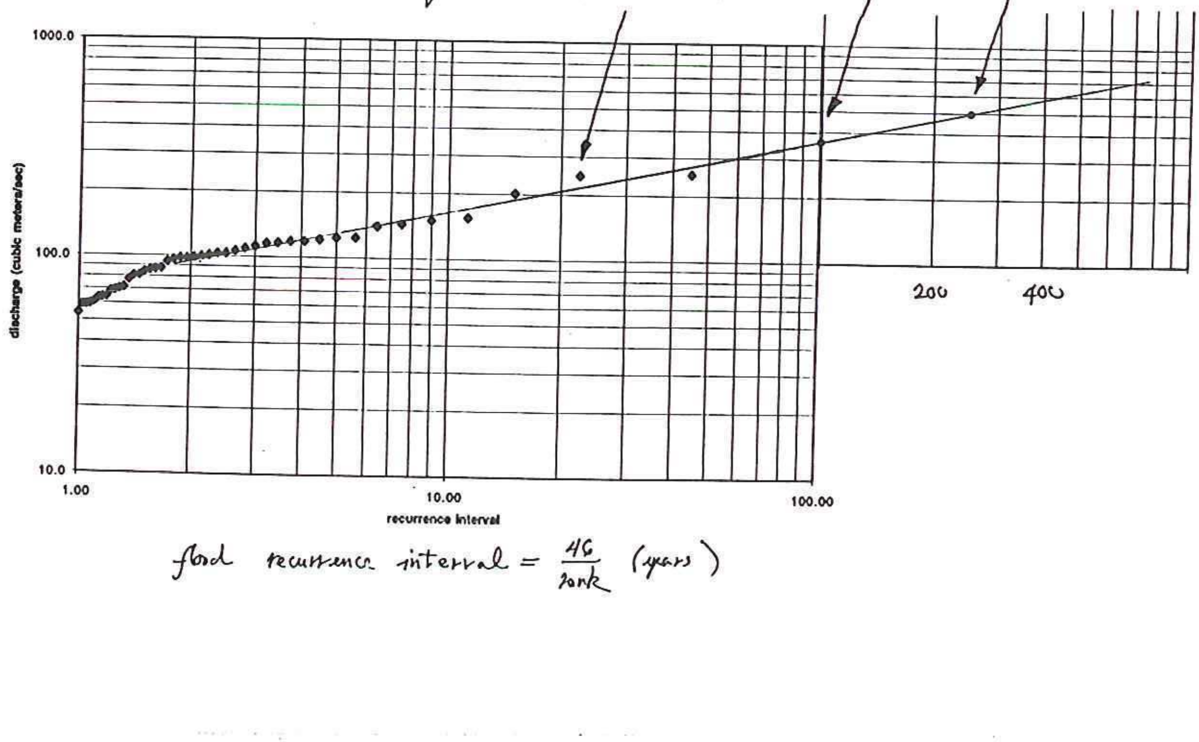
1954 to 1996 — 43 years of data



1954 – 1999 – 46 years of data

Flood forecasting – by extrapolation

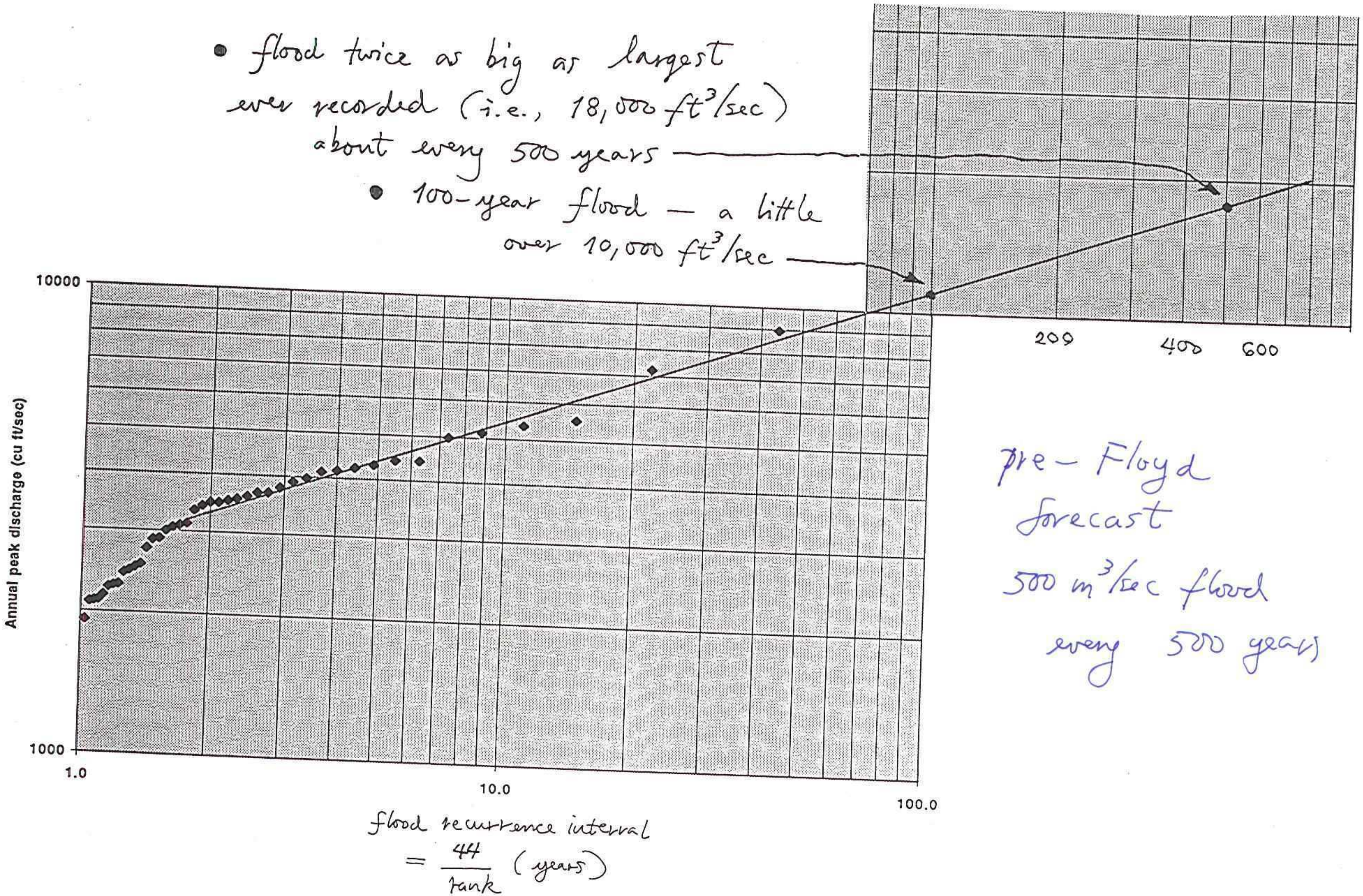
- flood twice as big as largest ever recorded (i.e., 500 m^3/sec) about every 250 years
- 100-year flood (about 360 m^3/sec)
- Hurricane Floyd 1999 (250 m^3/sec)



flood recurrence interval $= \frac{46}{\text{rank}}$ (years)

Flood forecasting — by extrapolation

- flood twice as big as largest ever recorded (i.e., 18,000 ft^3/sec) about every 500 years
- 100-year flood — a little over 10,000 ft^3/sec



pre-Floyd forecast
500 m^3/sec flood
every 500 years