Population Growth

An excellent reference: *How Many People Can the Earth Support?* Joel E. Cohen (1995)

Let N_t = number of people living on ⊕ at time t

$$N_{t+1} = N_t + B_t - D_t$$

B_t: no. of births; D_t: no. of deaths

Exponential growth — suppose that

$$B_t = b N_t, \qquad D_t = d N_t$$

b: per capita birth rate; d: per capita death rate

Then $$N_{t+1} = (1 + b - d) N_t = (1+r) N_t$$

where $r = b - d$ is the population growth rate

The solution to the difference equation

$$\boxed{\begin{aligned} N_{t+1} &= (1+r) N_t \quad \text{is} \\ N_t &= (1+r)^t N_0 \quad \star \end{aligned}}$$

This is of course the familiar compound interest formula. I shall refer to $\star$ as banker's notation. A scientist or demographer would instead write

$$N_t = N_0 e^{t \ln(1+r)}, \quad \text{or}$$

$$\boxed{N_t = N_0 e^{\lambda t} \quad \text{where} \quad \lambda = \ln(1+r)}$$

For small growth rates ($r \ll 1$):

$$\lambda = \ln(1+r) \approx r$$

For example, $\ln(1.05) = 0.04879 \approx 0.05$

scientist's rate ↗ banker's rate ↗

On a log-linear plot:

$$\boxed{\ln N_t = \ln N_0 + \lambda t} \leftarrow r \text{ is the slope}$$

Doubling time (analogous to half life of radioactive or topographic decay)

$$e^{\lambda t} = 2$$

$$\boxed{t_{double} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

Suppose now that the growth rate is a function of time — no longer constant

$$\boxed{N_{t+1} = (1 + r_t) N_t}$$

r_t time-dependent rate

In that case:

$$N_t = \underbrace{(1+r_0)(1+r_1)\cdots(1+r_{t-1})}_{\text{product of } t \text{ terms}} N_0$$

Taking the logarithm gives:

$$\ln N_t = \ln N_0 + \ln(1+r_0) + \ln(1+r_1) + \cdots \ln(1+r_{t-1})$$

$$= \ln N_0 + \lambda_0 + \lambda_1 + \cdots \lambda_{t-1}$$

$$\ln(N_t/N_0) = \lambda_0 + \lambda_1 + \cdots \lambda_{t-1}$$

$$\boxed{N_t = N_0 e^{\lambda_0 + \lambda_1 + \cdots + \lambda_{t-1}}}$$

If all the rates are the same ($\lambda_t = \lambda$) then $\lambda_0 + \lambda_1 + \cdots + \lambda_{t-1} = \lambda t$ and $N_t = N_0 e^{\lambda t}$ as before.

What is the <u>total</u> number of people that have <u>ever</u> lived upon the Earth?

It is <u>not</u> $N_{cum} = \sum_t N_t$

Rather, it is

$$\boxed{N_{cum} = \sum_t N_t / T_t}$$

where T_t is the life expectancy of the average person at time t.

The current rate of growth is

$$\lambda_{now} = 0.016 \quad (1.6\% \text{ per year})$$

$$T_{double} = 43 \text{ years}$$

$\lambda_{now} N_{now} = 100$ million new people added each year.

Current life expectancy T_{now}:

	women	men	average
developed	78 years	71 years	75 years
less developed	64 years	61 years	62 years
sub-Sahara Africa	55 years	52 years	53 years
world average	67 years	63 years	65 years

Birth rates are also commonly expressed as average number of babies born to a woman in her lifetime (aka fertility rate)

$$\frac{\#\text{ babies}}{\text{woman}} = b_t \cdot \left(1 + \frac{63}{67}\right) \cdot T_t$$

$$= \left(\frac{\text{births}}{\text{person yr}}\right)\left(\frac{\text{persons}}{\text{woman}}\right)\left(\frac{\text{yrs}}{\text{lifetime}}\right)$$

$$= (0.026)(1.9)(67)$$

$$= 3.3 \text{ babies/woman} \leftarrow \text{world-wide average}$$

Zero population growth requires

$$b_t = d_t$$

$$b_{now} = 2.6\%$$
$$d_{now} = 1.0\%$$

In a stationary steady-state population:

$$\boxed{b = d = 1/T}$$

This is just the residence time formula we have encountered before in this course:

$$T = \frac{N}{B}$$

N ← total population

B ← no. of people born per year

T ↑ residence time of a person in population pool

The zero population growth birth rate is not 2 babies/mother because the life expectancy of men is less than that of women:

$$\boxed{1 + \frac{67}{63} = 2.06 \text{ babies/mother for ZPG}}$$

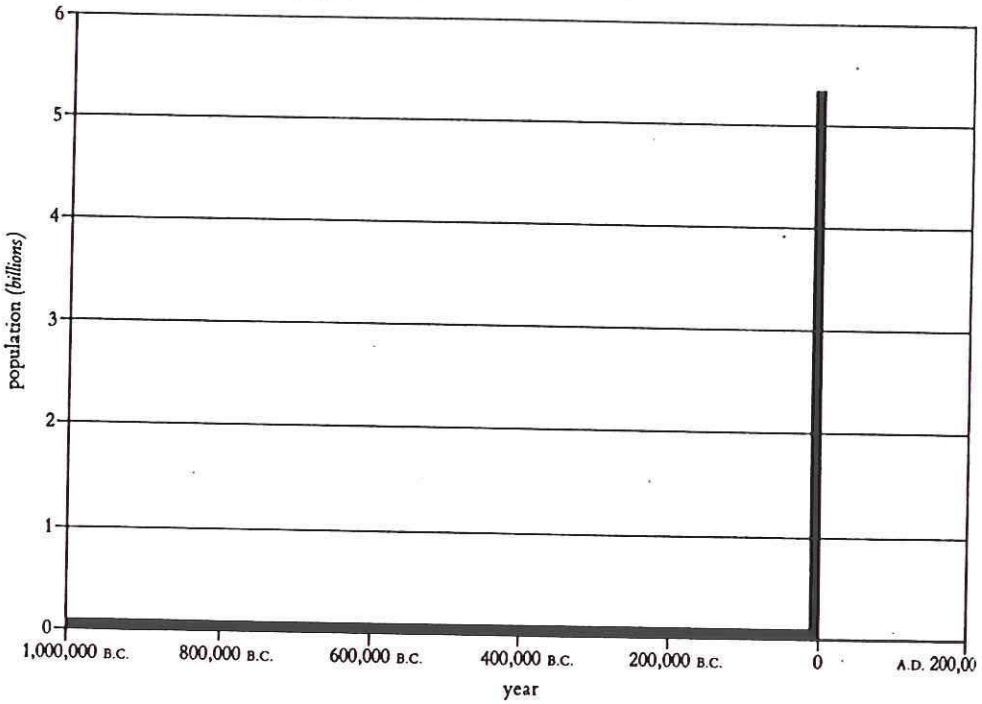


FIGURE 5.1 Estimated human population from a million years ago to the present.
SOURCE OF DATA: Appendix 2

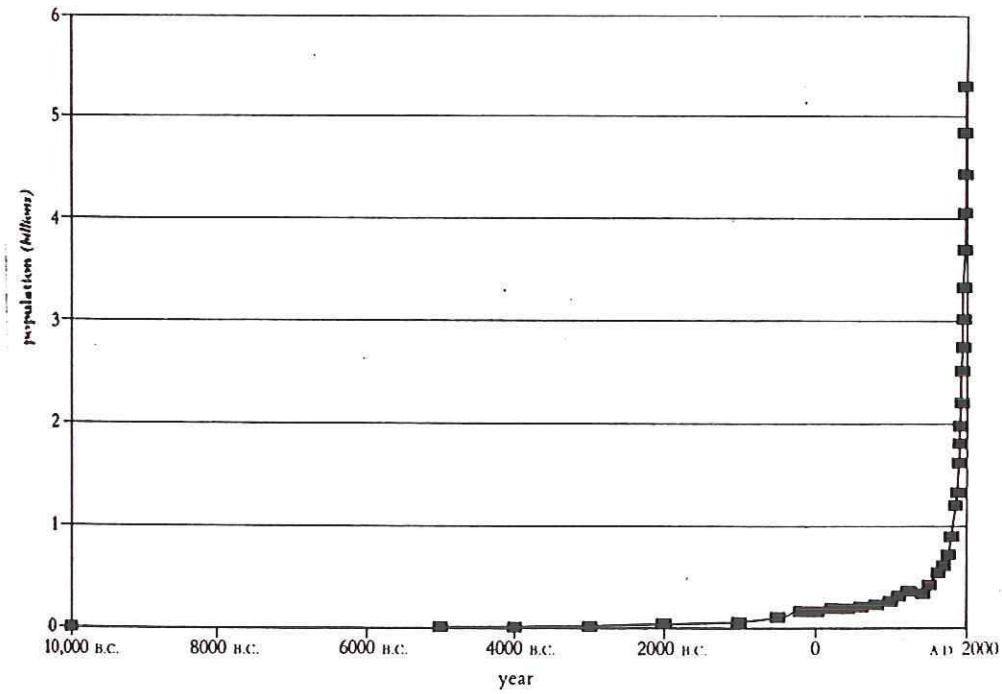


FIGURE 5.2 Estimated human population from the last ice age to the present.
SOURCE OF DATA: Appendix 2

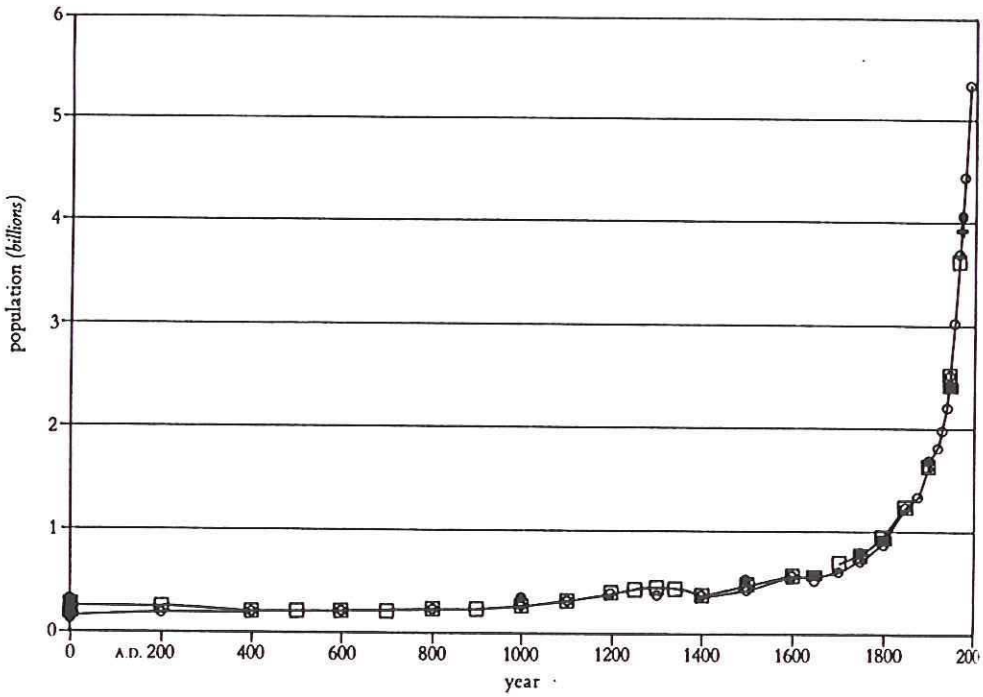


FIGURE 5.3 Estimated human population from A.D. 1 to the present. Different symbols represent estimates from different sources. SOURCE OF DATA: Appendix 2

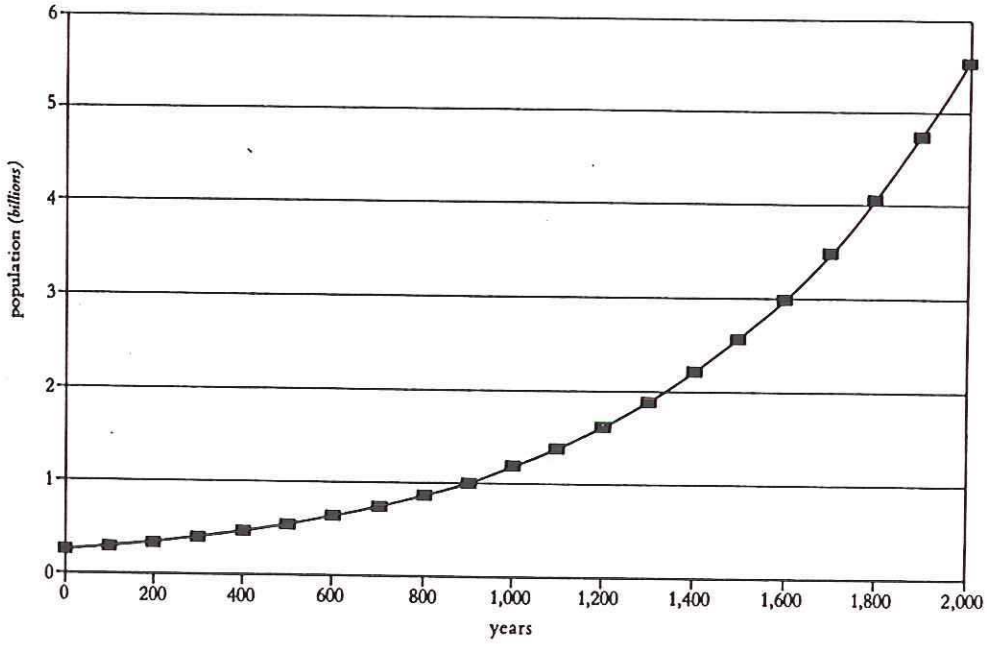


FIGURE 5.4 Hypothetical population growing exponentially from an initial value of 250 million people at 0.155 percent per year and reaching a population of 5.54 billion after 2,000 years. The population at year t is computed as $2.5 \times 10^8 \times (1.00155)^t$.

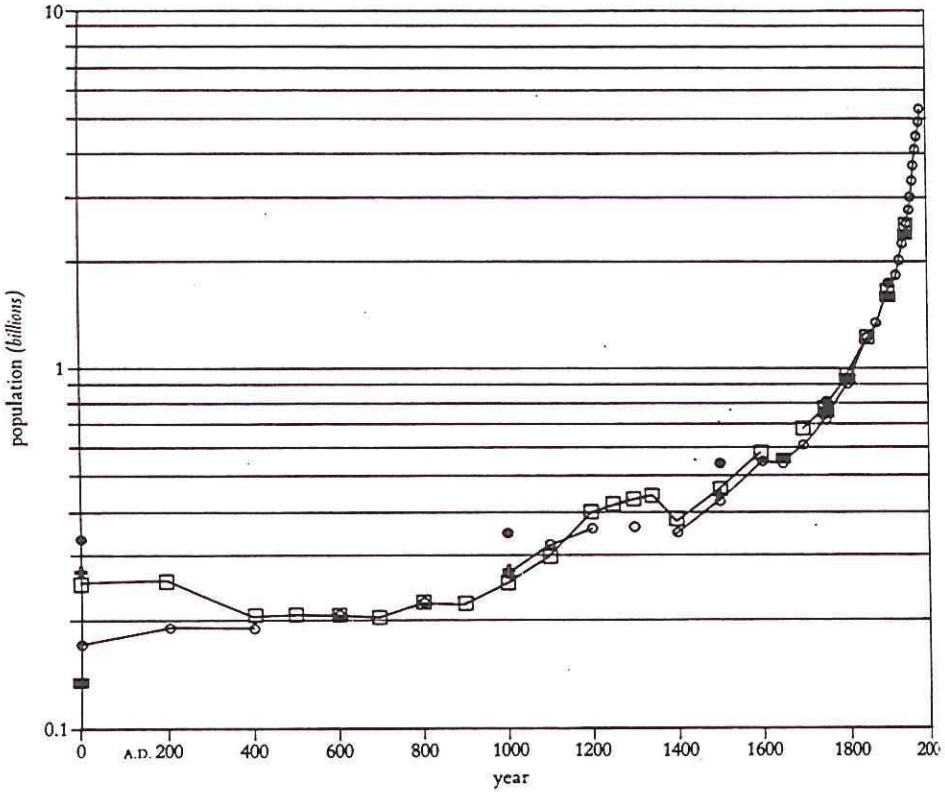


FIGURE 5.12 World population history for the last two millennia, with population plotted on a logarithmic scale. Different symbols represent estimates from different sources. SOURCE OF DATA: Appendix 2

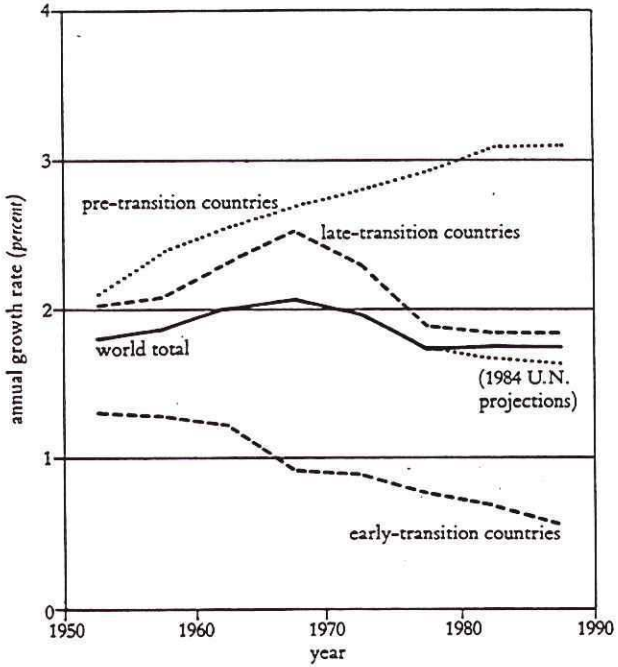


FIGURE 4.2 Global trends in the annual rate of population growth, 1950 to 1990, according to United Nations estimates. For the world total in 1990, the solid line shows the 1990 estimate and the dotted line just below it shows the growth rate projected in 1984; the anticipated decline in the population growth rate did not occur. SOURCE: Horiuchi (1992, p. 761, Fig. 1)

name of evolution	date in the middle	population *(billions)*	doubling time *(years)* before	doubling time *(years)* after
local agricultural	8000 B.C.	0.005	40,000–300,000	1,400–3,000
global agricultural	A.D. 1750	0.75	750–1,800	100–130
public health	1950	2.5	87	36
fertility	1970	3.7	34 (peak)	more than 40 (since 1990)

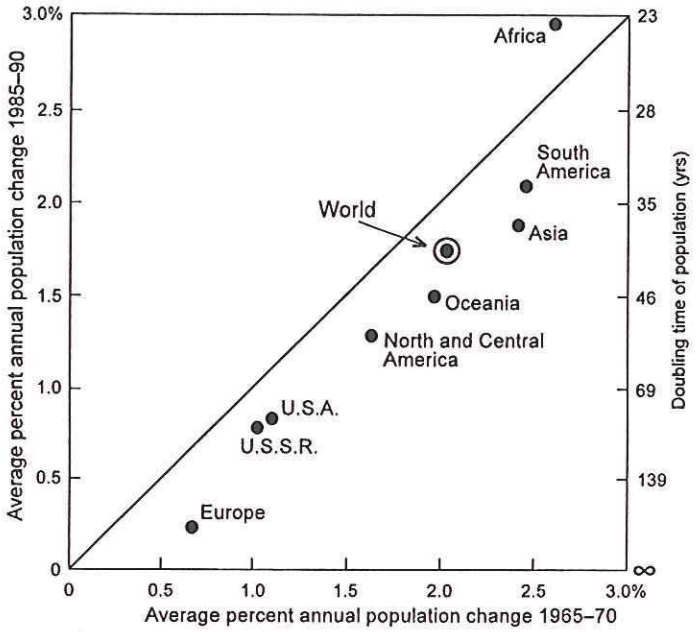


Figure 5.12. Comparison of annual population changes between 1985 and 1990 with those between 1965 and 1970 in several parts of the world.

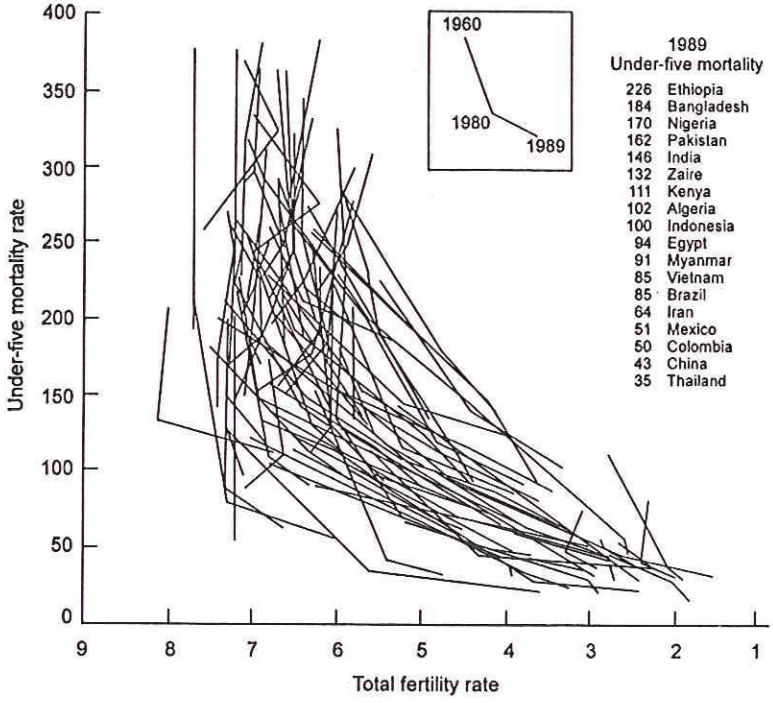


Figure 13.5. Child deaths and child births. Each line on the chart represents, for one developing country, the change in under-five mortality rate (USMR) and total fertility rate (TFR) over the period from 1960 to 1989. The intermediate point on each line represents data for the year 1980. On the right-hand side of the graph is shown the present under-five mortality rate of some of the most populous developing countries today. (Grant 1992); see also Brass and Jolly 1993, and Eubank and Gribble 1993)

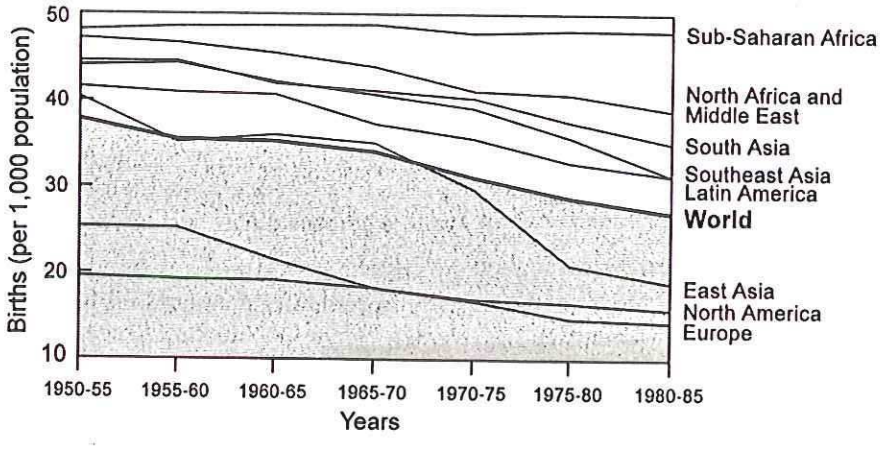


Figure 5.10. Birthrates in regions throughout the world have declined since the end of World War II. The only exceptions to this trend are birthrates in sub-Saharan Africa. Africa could account for nearly a quarter of the world's population by the late twenty-first century. (Caldwell and Caldwell 1990)

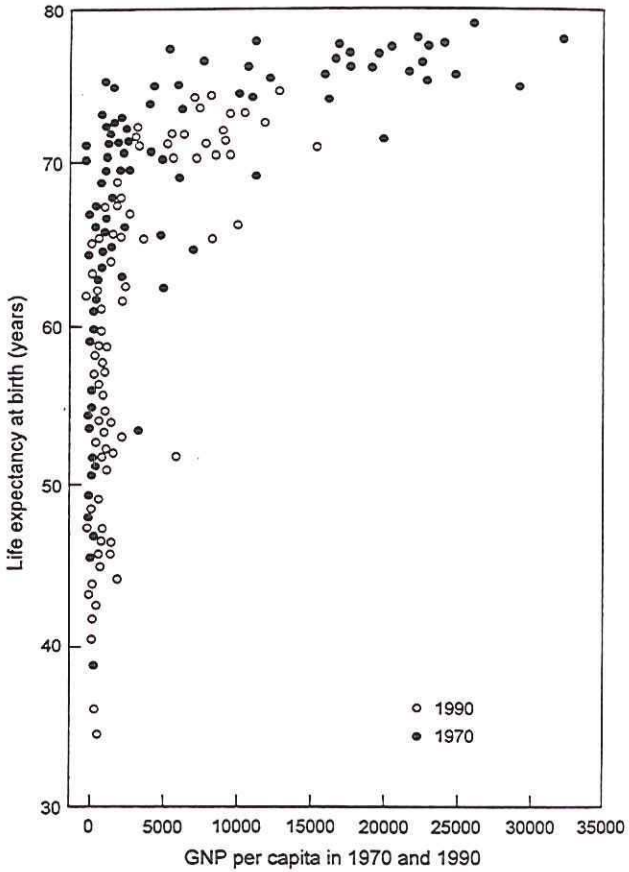


Figure 5.26. Life expectancy at birth in relation to income in countries for which data were available in 1970 and in 1990. Income is computed as gross national product per capita (GNPPC) and is plotted in 1990 U.S. dollars. (Wilkinson 1994)

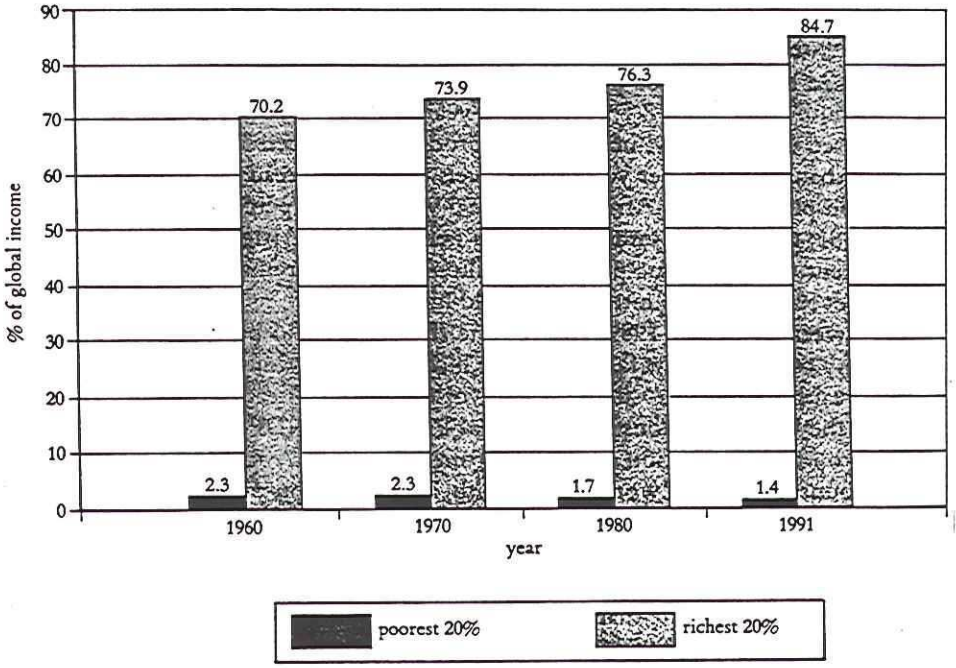


FIGURE 4.1 Fraction of global income received by the poorest 20 percent of people and the richest 20 percent of people from 1960 to 1991, according to the average gross national product per person of different countries. SOURCE: based on United Nations Development Program (1992, pp. 34, 36)

TABLE 8.1 Total fertility rates in 1990, by region

region	1990 total fertility rate	region	1990 total fertility rate
world	3.3		
U.N. group I		U.N. group II	
Europe	1.8	Africa	6.5
Northern America	1.8	China	2.5
Oceania	3.2	India	4.4
USSR	2.4	Latin America	4.1
		Other Asia	5.0

SOURCE: United Nations (1992a, p. 10)

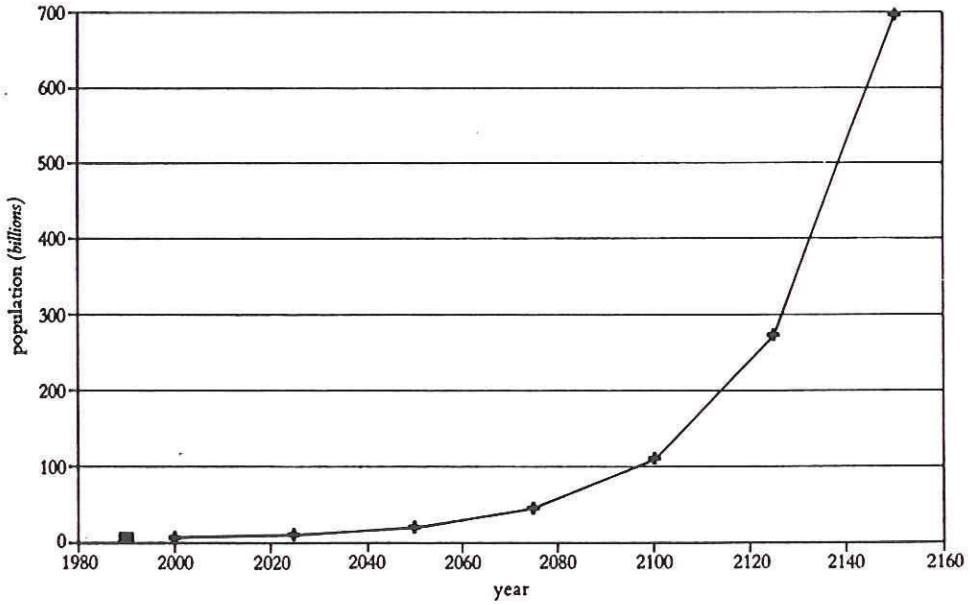


FIGURE 8.1 United Nations' projection of world population, assuming fertility remains constant at its 1990 levels in different regions. SOURCE: original figure drawn according to data of United Nations (1992a)

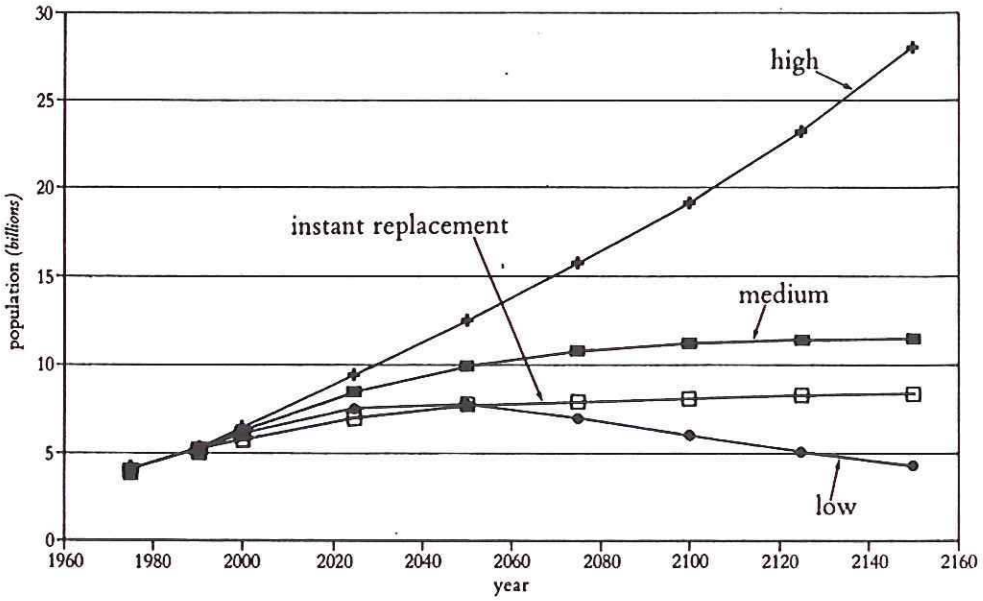
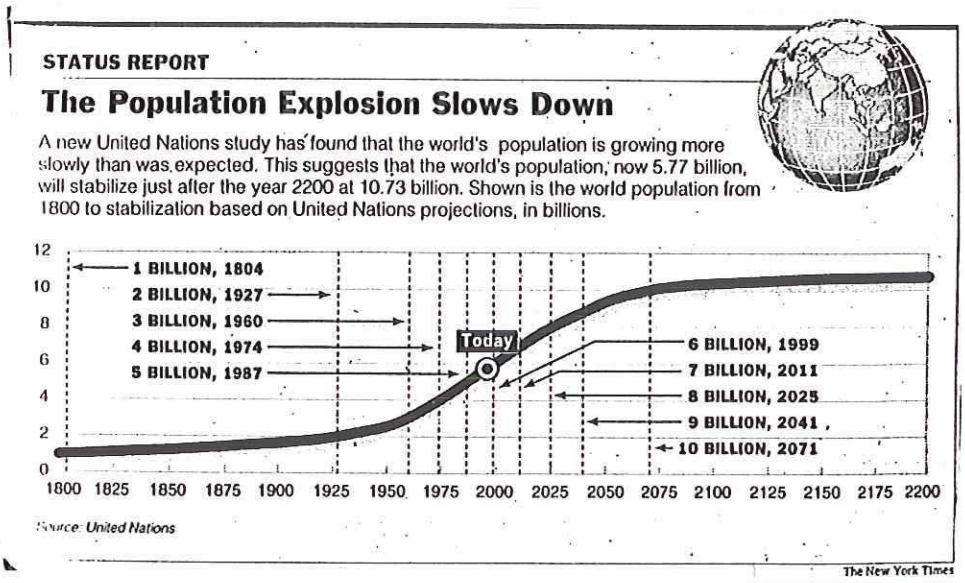


FIGURE 8.2 United Nations' projections of world population, according to high, medium, low and instant-replacement scenarios. SOURCE: original figure drawn according to data of United Nations (1992a)



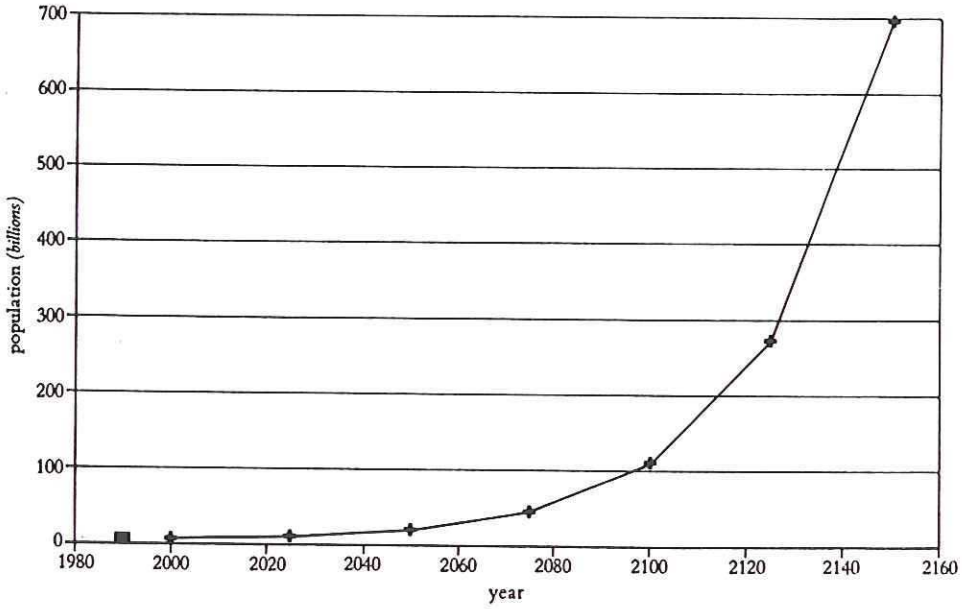


FIGURE 8.1 United Nations' projection of world population, assuming fertility remains constant at its 1990 levels in different regions. SOURCE: original figure drawn according to data of United Nations (1992a)

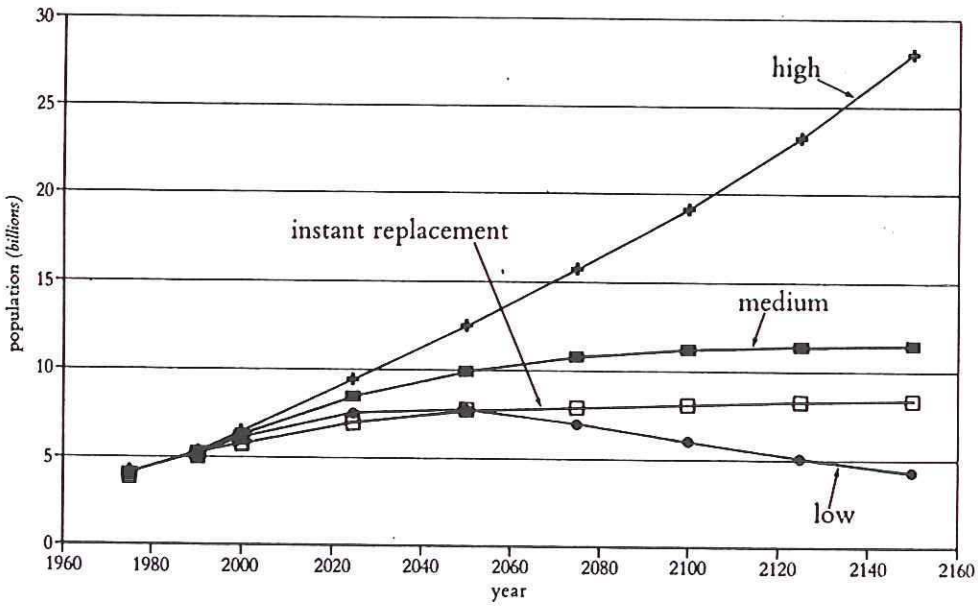


FIGURE 8.2 United Nations' projections of world population, according to high, medium, low and instant-replacement scenarios. SOURCE: original figure drawn according to data of United Nations (1992a)

Estimates of Past Human Population Sizes (Millions)

year (− means B.C.)	Deevey 1960	McEvedy and Jones 1978	Durand 1977 low	Durand 1977 high	Clark 1977	Biraben 1979	Blaxter 1986[a]	United Nations 1992a	Kremer 1993[b]
−1,000,000	0.125	—	—	—	—	—	—	—	0.125
−300,000	1	—	—	—	—	—	—	—	1
−25,000	3.34	—	—	—	—	—	—	—	3.34
−10,000	—	4.00	—	—	—	—	—	—	4.00
−8000	5.32	—	—	—	—	—	—	—	—
−5000	—	5.0	—	—	—	—	40	—	5.0
−4000	86.5	7	—	—	—	—	—	—	7
−3000	—	14	—	—	—	—	—	—	14
−2000	—	27	—	—	—	—	70[c]	—	27
−1000	—	50	—	—	—	—	100[d]	—	50
−500	—	100	—	—	—	—	—	—	100
−400	—	—	—	—	—	153	162	—	—
−200	—	150	—	—	—	225	231	—	150
1	133	170	270	330	256[e]	252	255	—	170
200	—	190	—	—	—	257	256	—	190
400	—	190	—	—	254[f]	206	206	—	190
500	—	190	—	—	—	207	—	—	—
600	—	200	—	—	237	208	206	—	200
700	—	210	—	—	—	206	207	—	—
800	—	220	—	—	261	224	224	—	220
900	—	240	—	—	—	222	226	—	—
1000	—	265	275	345	280	253	254	—	265
1100	—	320	—	—	—	299	301	—	320
1200	—	360	—	—	384	400	400	—	360
1250	—	—	—	—	—	417	—	—	—
1300	—	360	—	—	—	431	432	—	360
1340	—	—	—	—	378	442	—	—	—
1400	—	350	—	—	—	375	374	—	350
1500	—	425	440	540	427	461	460	—	425
1600	—	545	—	—	498	578	579	—	545
1650	545	545	—	—	516	—	—	—	545
1700	—	610	—	—	641	680	679	—	610
1750	728	720	735	805	731	771	770	—	720
1800	906	900	—	—	890	954	954	—	900
1850	—	1,200	—	—	1,190	1,241	1,241	—	1,200
1875	—	1,325	—	—	—	—	—	—	1,325
1900	1,610	1,625	1,650	1,710	1,668	1,634	1,633	—	1,625
1920	—	—	—	—	—	—	—	—	1,813
1925	—	2,000	—	—	—	—	—	—	—
1930	—	—	—	—	—	—	—	—	1,987
1940	—	—	—	—	—	—	—	—	2,213
1950	2,400	2,500	—	—	—	2,530	2,513	2,516	2,516
1955	—	—	—	—	—	—	—	2,752	—
1960	—	—	—	—	—	—	—	3,020	3,019
1965	—	—	—	—	—	—	—	3,336	—
1970	—	—	—	—	—	3,637	—	3,698	3,693
1975	—	3,900	3,950	4,050	—	—	—	4,079	—
1980	—	—	—	—	—	—	4,415	4,448	4,450
1985	—	—	—	—	—	—	—	4,851	—
1990	—	—	—	—	—	—	—	5,292	5,333

[a] Blaxter's estimate "derives from" those of Biraben (1979) and the United Nations (Blaxter 1986, p. 12), but minor differences from Biraben's figures are not explained.

[b] Kremer's estimate is based on Deevey (1960) up to −25,000, on McEvedy and Jones (1978) from −10,000 to 1900 and on various sources after 1900.

[c] Blaxter's (1986, p. 13) estimate for 1600 B.C. is shown on the line for 2000 B.C.

[d] Blaxter's (1986, p. 13) estimate for 800 B.C. is shown on the line for 1000 B.C.

[e] Clark's (1977, p. 64) estimate for A.D. 14 is shown on the line for A.D. 1.

[f] Clark's (1977, p. 64) estimate for A.D. 350 is shown on the line for A.D. 400.

SOURCES: Deevey (1960); McEvedy and Jones (1978); Durand (1977); Clark (1977); Biraben (1979); Blaxter (1986); United Nations (1992a); Kremer (1993)