Heat flow at the ⊕'s surface

Heat is a form of energy. It is the macroscopic manifestation of the energy of molecular motion.

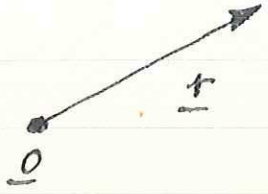
Heat tends to move from hot places to cold places.

Three mechanisms of heat transfer: conduction, convection and radiation.

Conduction dominant in lithosphere although as we shall see heat transfer by hydrothermal convection is also important in the oceans (and in hot spring areas on land).

We shall however study heat flow by conduction first: same mechanism that makes handle of a cast iron skillet get hot.

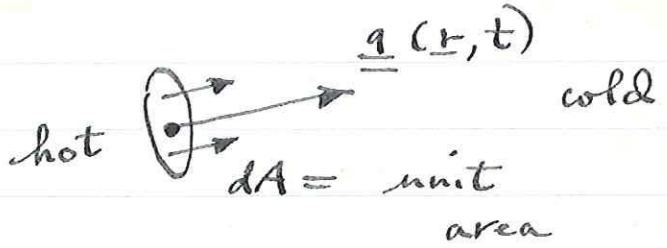
Consider a body: $\underline{r}$ = position vector



If some parts of the body are hotter than others, heat flow will occur to equalize temp.

Heat flow or flux is a vector field in the body, call it $\underline{q}(\underline{r})$. To define $\underline{q}(\underline{r})$ must specify direction and magnitude.

1. direction: direction heat is flowing
2. amount of energy flowing through a unit area per second: magnitude



Units of $\underline{q}(\underline{r},t)$: SI units $\frac{J}{m^2\,sec}$

or $\frac{W}{m^2}$.

An older unit for heat (used before it was realized that heat was just a form of energy) the calorie = 4.184 J

A unit frequently used in geophysics:

> 1 HFU (heat flow unit)
> = 1 μcal / cm^2-sec
> = 41.84 mW / m^2

This a convenient unit because a typical heat flux thru $\oplus$'s surface is a few HFU.

The average heat flow escaping from the $\oplus$ is 80 ± 8 mW/m^2 ~ 2 HFU (Davies, 1980 RGSP). Sclater et al. say 70 mW/m^2. This cited in Turcotte & Schubert

$[q] = 80 \text{mW/m}^2$

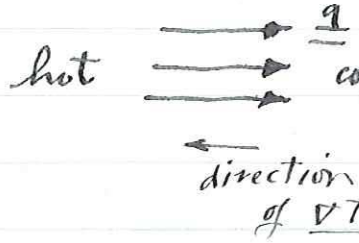
Fourier's "law" of heat conduction relates the heat flux at a point to the temp. gradient

$$\underline{q}(\underline{r}, t) = -\kappa(\underline{r})\, \underline{\nabla T}(\underline{r}, t)$$

Says heat flow is in direction of temp. grad. and $\propto$ to it. Not really a law like, say, $E = mc^2$, may break down in extreme situations, e.g.

during an explosion.

– sign so that flow from hot to cold



κ is positive

κ, proportionality const. called <u>thermal conductivity</u>, units cal / cm – sec – °C. or W / m – °C.

<u>Conductivity</u> a material property, good conductors (Cu, Fe) have large κ, poor conductors (home insulation, e.g.) have low conductivity. Also κ can be temperature dependent.

One measures κ by applying heat (electrical resistance heating) to one side of a small sample and detecting the thermal response on the other side.

Example: high T conductivity of <u>olivine</u> (dominant constituent of oceanic lithosphere) from Schatz + Simmons <u>JGR</u> 1972 shown.

Typical value is $\kappa \sim 0.01$ cal/°C.-cm-s
Parsons and Sclater adopt a somewhat lower value * as an average of for upper ~100 km of mantle, viz.

$$\kappa \approx 0.0075 \text{ cal/°C.-cm-sec}$$

This is about 20-30 times lower than a good conductor such as Cu or Fe. It is a common observation that rocks are better insulators than steel plates.

The interior of the ⊕ is hotter than the surface and as a result heat is flowing out. The heat flow at ⊕'s surface is given by

$$q = \kappa (dT/dz)$$

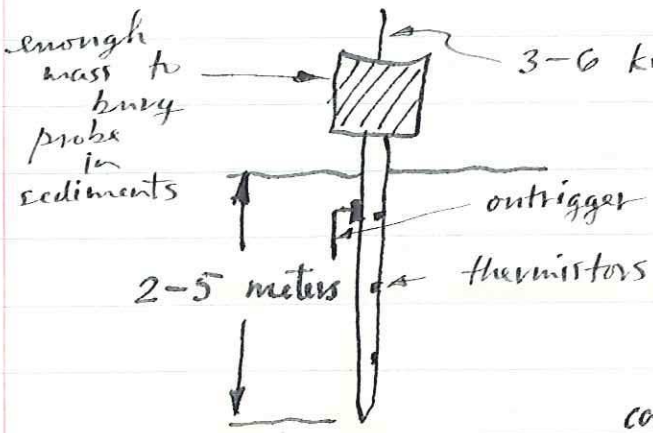
To measure the heat flow we measure dT/dz and multiply by κ.

First measurements were made on land down boreholes. Preferably these should be as deep as possible to avoid surface disturbances. Also heat transfer by ground water convection can and does plague many measurements. Perturbations associated with last ice age also important.

First measurements at sea 1956 Bullard, Maxwell and Revelle. Previously thought to be very difficult. Now commonplace, easier and more reliable and cheaper than land measurements which require deep (≫ 100 m) boreholes. Ocean floor very stable thermally.

on land must be careful to avoid circulation of drilling fluid.

Seafloor almost everywhere covered by soft sediments. Method, oceanic heat flow probes



To measure κ of sediments can core or add an electrically heated outrigger

Or can core seds and bring back to lab.

κ of ocean-floor muds depends on H_2O content (must not allow to dry out before measuring κ).

Typically $\kappa_{mud} \sim 0.004$ cal/°C.cm s (or, equiv., $\sim 2 \frac{W}{m\,°C}$)
which means $dT/dz \sim 0.05°$ per meter or $0.1°$ for a 2 m probe. This not difficult to measure because of thermal stability of ocean floor.

First oceanic heat flow measurement eagerly awaited. Expected to be much lower than on land because radioactive heat productivity of basalts is ~ 30–40 times less than granites.

Famous story "I put it in the fixer first". Result: average oceanic and continental heat flows roughly equal. Now thought that average oceanic heat flux about 70% greater (including hydrothermal fluid circulation) than average continental.

Tendency for higher heat flow over and near mid-ocean ridges an early discovery. Early evidence for seafloor spreading.

Now many 1000's of oceanic heat flow measurements. General characteristics:

1. very scattered in young crust less than, say, 30–40 m.y. includes high values up to 5–6 HFU.
2. less scatter beyond 30–40 m.y. typically about 1 HFU.

Example: compilation of data from Indian Ocean by Anderson et al. JGR (1977). Age determined from identification of magnetic anomalies, locations shown in map. Heat flow in HFU plotted as a function of age in Fig. 9A.

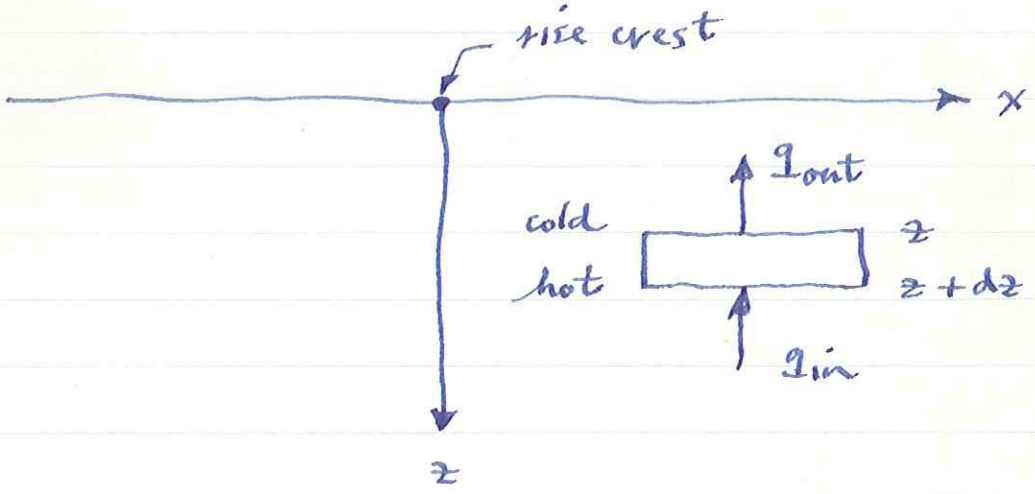
When sorted according to "reliability" using various criteria (e.g. uniformity of sediment cover) and averaged and smoothed the general trend shown in Fig. 9B emerges. Much local variability (between nearby stations) in region < 30–40 m.y., heat flow about 1 HFU in old basins.

We shall now investigate the explanation of heat flow variation + subsidence of topog. + geoid.

Thermal evolution of the oceanic lithosphere

We need to work out the equation which describes the cooling of the lithosphere.

To do this we shall neglect the surface topography and consider the cooling of a half-space. This can be justified since the cooling extends down to depths $\gg$ the variation in topography.



1m x 1m Consider a small block of material 1 cm x 1 cm laterally between z and $z + dz$ in depth.

We'll assume there is no heat flow through the sides of the block.

This can be justified since the isotherms are nearly flat except very near the rice crest.

Let U be the total heat energy content in the box (the internal energy, to be more precise).

The rate of change of U with time is given by

$$\frac{dU}{dt} = q_{in,\,bottom} - q_{out,\,top}$$

$[q]$ = ergs/cm^2-sec or J/m^2-sec or W/m^2
$[U]$ = ergs or calories or J

But from Fourier's law

$$q_{out} = \text{[redacted]}\; k\,(dT/dz)_{z}\; \text{[redacted]}$$

$$q_{in} = \text{[redacted]}\; k\,(dT/dz)_{z+dz}$$

Thus

$$dH/dt = k\left[(dT/dz)_{z+dz} - (dT/dz)_z \right]$$

Now we must relate dH/dt to the rate of change of temperature.

The quantity which does this is called the specific heat (at const. pressure) c_p.

By defn, c_p = the amount of heat energy required to raise the temp. of one gm. of material by 1°C.

The specific heat of water is

$$c_p(H_2O) = 1 \text{ cal / gm °C; at STP}$$

This is the definition of the calorie. In SI units

$$c_p(H_2O) = 4184 \text{ J / kg °C.}$$

The specific heat of a typical igneous rock is about one-quarter this. Sclater and Parsons (1977) adopted value:

$c_p = 1170 \frac{J}{kg°C}$ $c_p = 0.28$ cal / g °C.

The specific heat of all inorganic solids at room temp or above is about the same if expressed in terms of cal / mole °C.

The "law" of Dulong and Petit:

c_p (solids) ~ 6 cal / mole °C.

i.e. mol. wt. per atom

The average molecular weight of olivine is $M \sim 20$, which gives

$c_p \sim 0.3$ cal / g °C.

$Mg_2 Si O_4$

$48 + 28 + 64 = 140 \div 7 = 20$

The "law" of Dulong and Petit is very well understood in terms of the energy of molecular excitation.

We thus have

$$dU/dt = \rho \cdot c_p \cdot 1^2 \cdot dz \cdot (dT/dt)$$

$$\text{cal/sec} = \frac{\text{gm}}{\text{cm}^3} \cdot \frac{\text{cal}}{\text{gm °C.}} \cdot \text{cm}^3 \cdot \frac{\text{°C.}}{\text{sec}}$$

units check.

$$\rho c_p \frac{dT}{dt} = \kappa \underbrace{\left[\frac{(dT/dz)_{z+dz} - (dT/dz)_z}{dz} \right]}$$

looks like derivative

Now taking limit as $dz \to 0$ we get

$$\rho c_p \frac{dT}{dt} = \kappa \frac{d^2T}{dz^2}$$

Actually T is a function of both z and t and the usual notation for the above derivatives is the partial derivative

$$\rho c_p \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

In the above argument we have neglected the possibility of radioactive heating. The decay of K, U and Th in the rocks can also increase the heat content of our block.

Let ~~k~~ A = heat productivity by radioactive heating, units cal/cm^3-sec (amount of energy or heat added per second). Better to call this A for later agreement.
This can be measured by measuring radioactivity content of rocks.

Clearly ~~h~~ A must be added to our eqn as follows:

$$\rho C_p \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} + \cancel{h}\ A$$

call this A instead

$\rho C_p \frac{\partial T}{\partial t}$	$\kappa \frac{\partial^2 T}{\partial z^2}$	~~h~~ A
total rate of change of heat energy (cal/cm³-sec)	change due to heat flow thru top + bottom	change due to radio-active heating

This is partial differential equation: called the 1-d heat flow (or diffusivity) eqn.

In oceanic crust and lithosphere A ~~h~~ is quite small and can be neglected: Table 7.2 of Stacey, page 186.

material	~~h~~ A instead (10⁻⁶ W/m³)
due to U, Th, K granites	2.8
large radius tholeitic basalts	0.079 (35 times smaller)
elements dunites, peridotites	0.005 (600 times smaller)

Contribution from radioactive decay more important in granitic cont. crust,

negligible in mantle and in oceanic crust.

Rewrite equation as

$$\boxed{\partial T/\partial t = k\, \partial^2 T/\partial z^2} \quad *$$

where $\boxed{k = \kappa/\rho c_p$, the thermal diffusivity}

Units of k: $\dfrac{\text{cal}}{^\circ\text{C. cm sec}} \times \dfrac{\text{cm}^3}{\text{gm}} \times \dfrac{\text{gm}\ ^\circ\text{C}}{\text{cal}}$

$$= \frac{\text{cm}^2}{\text{sec}}$$

This agrees with units in equation:

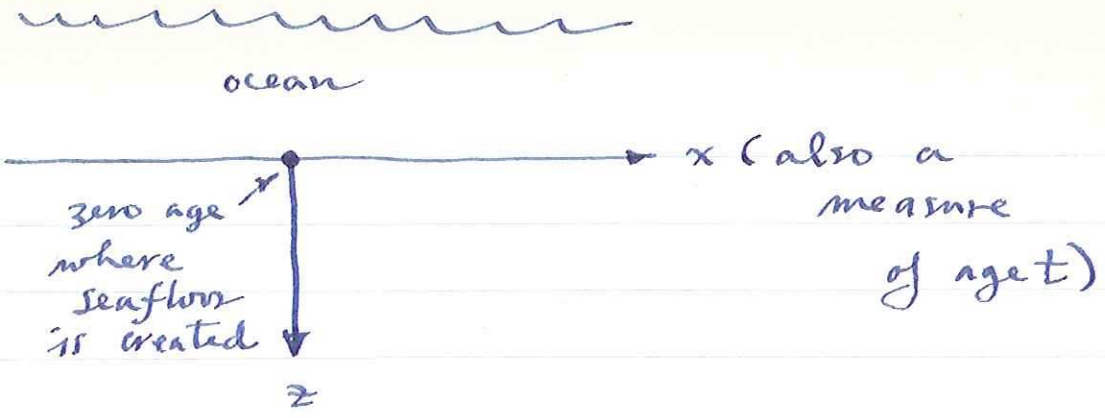
$$\frac{^\circ\text{C.}}{\text{sec}} = \frac{\text{cm}^2}{\text{sec}} \times \frac{^\circ\text{C.}}{\text{cm}^2}$$

Typical value $k \sim 0.008$ cm^2/sec

i.e. 0.8 mm^2/sec

The p.d.e. * is just about the world's simplest p.d.e. Whole books have been written on how to solve it in more complicated geometries.

We know we need boundary and initial conditions to be specified.



We specify $T(0, t) = 0$ temp on sea bottom just 1° or 2° above 0°C.

Also $T(\infty, t) = T_m$, ambient temperature in mantle, much hotter of order 1300°C.

Also $T(z, 0) = T_m$ temp at zero age is T_m at all depths.

This is known to be just the right number kind of b.c. and i.c. for the heat flow eqn — a parabolic p.d.e.

Instead of stopping to learn how to solve p.d.e.'s let us just verify the solution.

Consider, then, the solution

$$T(z,t) = \frac{2}{\sqrt{\pi}} T_m \int_0^{z/\sqrt{4kt}} e^{-u^2}\, du \qquad **$$

Let us verify this is the solution:

1. $T(0,t) = \int_0^0 du$ vanishes, satisfies b.c. on top.

2. $T(\infty,t) = T(z,0)$

$$= \frac{2}{\sqrt{\pi}} T_m \underbrace{\int_0^\infty e^{-u^2}\, du}$$

definite integral has value $\sqrt{\pi}/2$.

$= T_m$. Thus temp at 0 age and at ∞ depth is that of ambient mantle.

3. $$\partial T/\partial t = \frac{2}{\sqrt{\pi}} T_m\, e^{-z^2/4kt} \left(-\tfrac{1}{2} t^{-3/2}\right) \frac{z}{\sqrt{4k}}$$

$$= -\frac{T_m}{2\sqrt{\pi}} e^{-z^2/4kt} \left(\frac{z}{\sqrt{k}\, t^{3/2}}\right)$$

$$\partial^2 T/\partial z^2 = \frac{2}{\sqrt{\pi}} T_m e^{-z^2/4kt} \frac{1}{\sqrt{4kt}} \left(-\frac{2z}{4kt}\right)$$

$$= -\frac{T_m}{2\sqrt{\pi}} e^{-z^2/4kt} \left(\frac{z}{\sqrt{k}\, t^{3/2}}\right) \frac{1}{k}$$

Thus $\partial T/\partial t = k\, \partial^2 T/\partial z^2$, so ** is our solution.

Conventional notation: define the so-called <u>error function</u> (arises in theory of Gaussian errors — <u>not</u> named after the famous mathematician Error) Wolfgang Windgassen Error

$$\boxed{\operatorname{erf} q = \frac{2}{\sqrt{\pi}} \int_0^q e^{-u^2}\, du}$$

Then the solution can be written

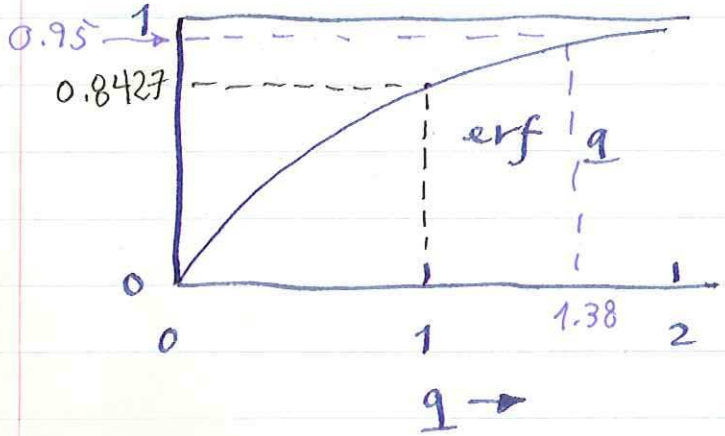
$$\boxed{T(z,t) = T_m \operatorname{erf}\left(z/\sqrt{4kt}\right)}$$

This gives temp. at time t and depth z in a cooling half-space.

Properties of erf:

$$\operatorname{erf} 0 = 0$$
$$\operatorname{erf} \infty = 1$$

Plot of erf looks like

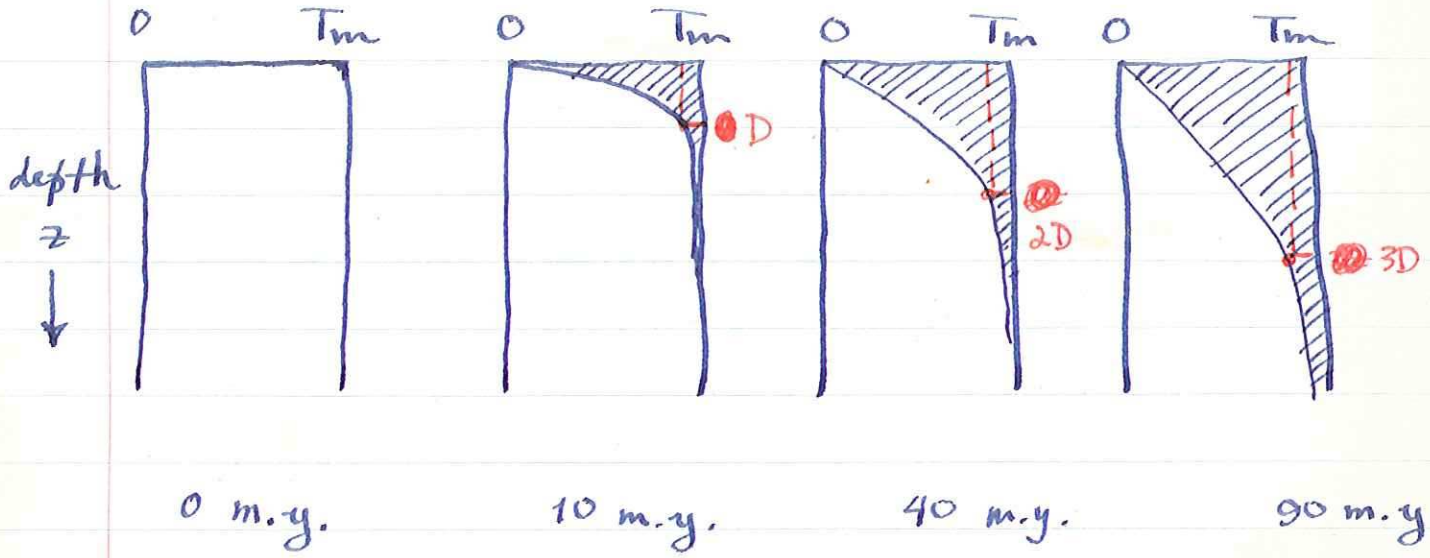


For example

$$\operatorname{erf} 1 = 0.8427$$
$$\operatorname{erf} 1.38 = 0.95$$

Note that T depends only on the combination $z / \sqrt{4kt}$. This means that at 40 m.y. the cooling has gone down 2 times as far as at 10 m.y. and at 90 m.y. it will have gone down 3 times as far.

Plots of T vs. z at various times thus look like red line denotes $T = 0.95 T_m$



The cooling proceeds like $\sqrt{t}$.

At what depth $\not{z}^D$ is $T = 95\%$ of T_m?

Answer:

$$\not{z}_D / \sqrt{4kt} = 1.38$$

$$\not{z}_D = 2.76 (kt)^{1/2}$$

We just decide to take this as depth to which appreciable cooling has penetrated.

If $k = 0.008$ cm^2/sec

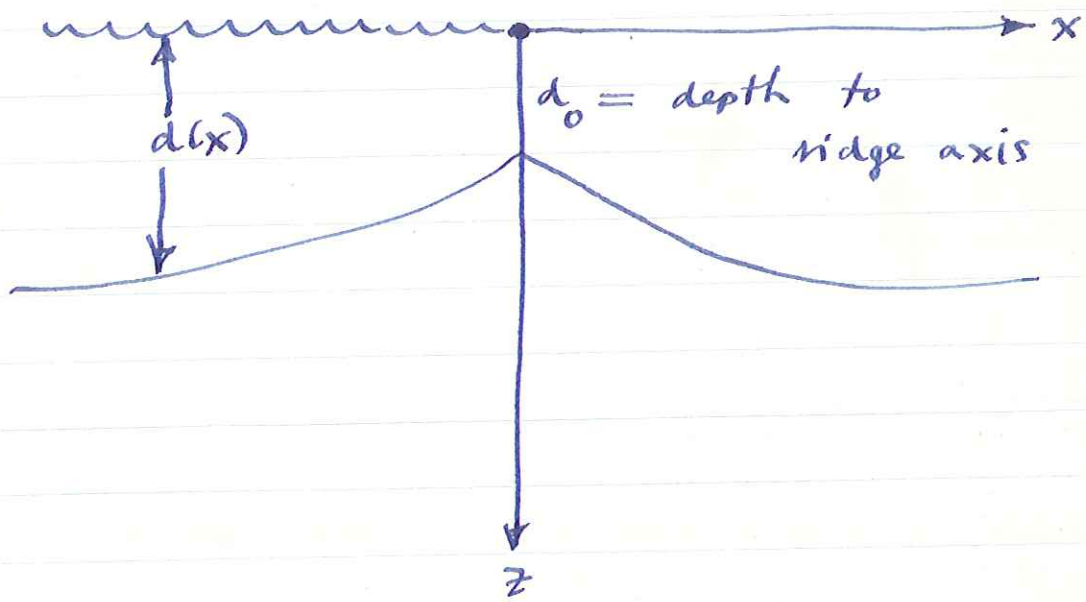
$$\not{z}_D = 13.9\, t^{1/2}$$ where t in m.y. and $\not{z}^D$ in km

Isostatic geoid anomalies over the oceanic lithosphere

The oceanic lithosphere as it spreads away from mid-ocean ridge axes cools off and subsides due to thermal contraction.

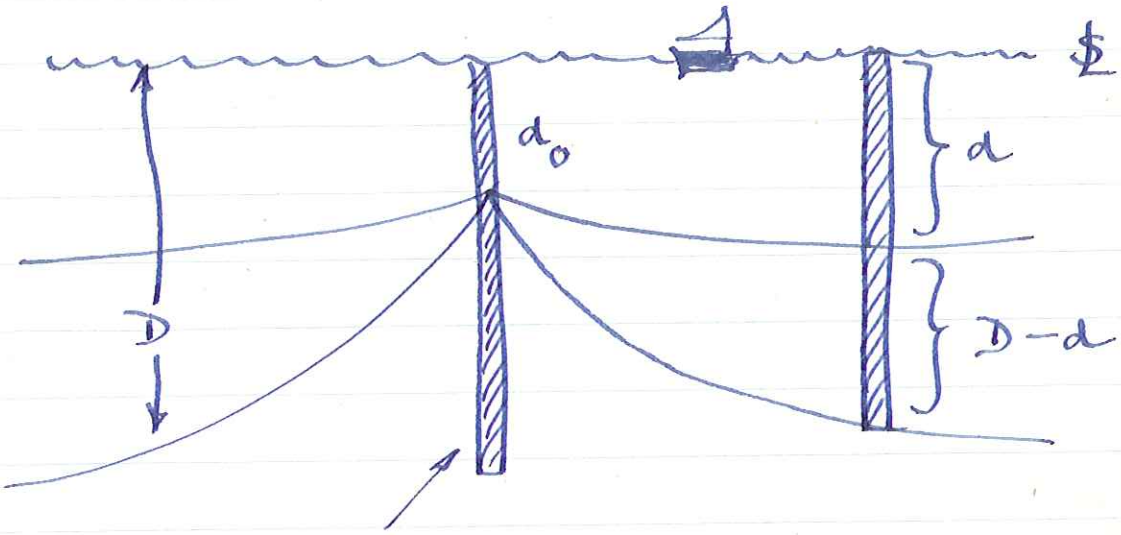
We wish to calculate expected geoid height anomaly due to this phenomenon.

Since isostasy involves only uppermost regions of $\oplus$, we consider a locally flat model:



Sea H_2O density $\rho_w = 1.025 \sim 1$ gm/cm^3 ← greater than 1 (salty)

Standard column at ridge axis



density of ambient hot mantle = $\rho_m = 3.33\ g/cm^3$

Significant cooling has occurred down to a depth D (compensation depth). We'll eventually let $D \to \infty$.

Two approximations will be made:

1. $d \ll D$, subsidence $\ll$ depth down to which cooling has occurred.

2. $D \ll L$, depth scale $\ll$ horizontal scale L over which $\exists$ significant lateral variations.

These 2 approx. are valid since, as we shall see, $d \sim 4$ km, $D \sim 100$ km and $L \sim 1000$ km. ($d - d_0 \approx 2$ km)

The approx. $D \ll L$ limits consideration to features (such as subsidence away from ridge) of broad horizontal scale (for which local isostasy a good approx.)

Anomalous density in $d < z < D$:
$\delta\rho(x, z)$ so total density is
$\rho_m + \delta\rho(x, z)$
$\delta\rho$ positive due to thermal contraction.

What is condition for isostatic balance?
Two shaded columns must have equal mass / cm^2.
Consider 1 cm^2 cross-sectional area.

Ridge column mass $= \rho_w d_0 + \rho_m (D - d_0)$

Flank column mass $= \rho_w d$

$$+ \int_d^D (\rho_m + \delta\rho)\, dz$$

$$= \rho_w d + \rho_m (D - d) + \int_0^D \delta\rho \, dz$$

can replace ↑ by zero with second order error as both d and $\delta\rho$ first order.

$$= \overbrace{\rho_w d_0 + \rho_m (D - d_0)}^{\text{same as ridge column}} + \underbrace{(\rho_w - \rho_m)(d - d_0) + \int_0^D \delta\rho \, dz}_{\text{must vanish}}$$

$$\boxed{(\rho_m - \rho_w)(d - d_0) = \int_0^\infty \delta\rho(x,z) \, dz}$$

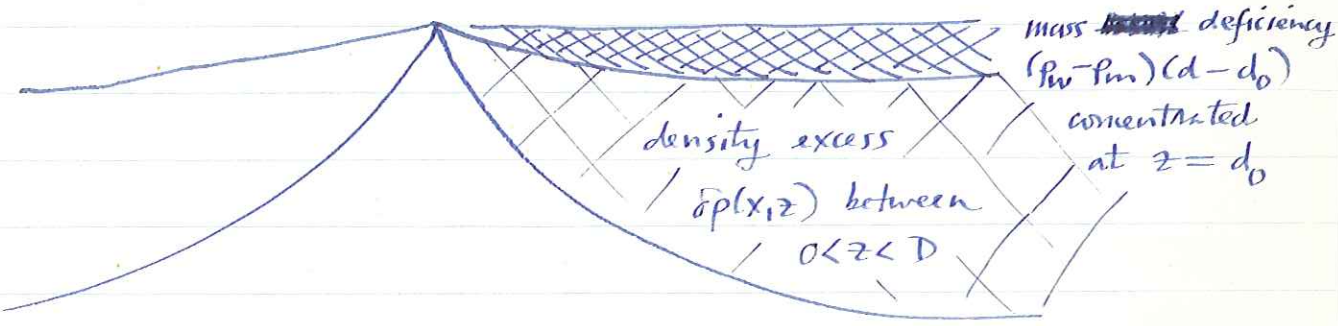
Relates <u>subsidence</u> $d(x) - d_0$ away from ridge to <u>anomalous</u> density in cooling lithosphere.

Now what is the geoid signal? A slightly tricky argument is required to find this. We seek the anomalous grav. potential $V(x)$ on $\mathcal{L}$, the level $z = 0$.

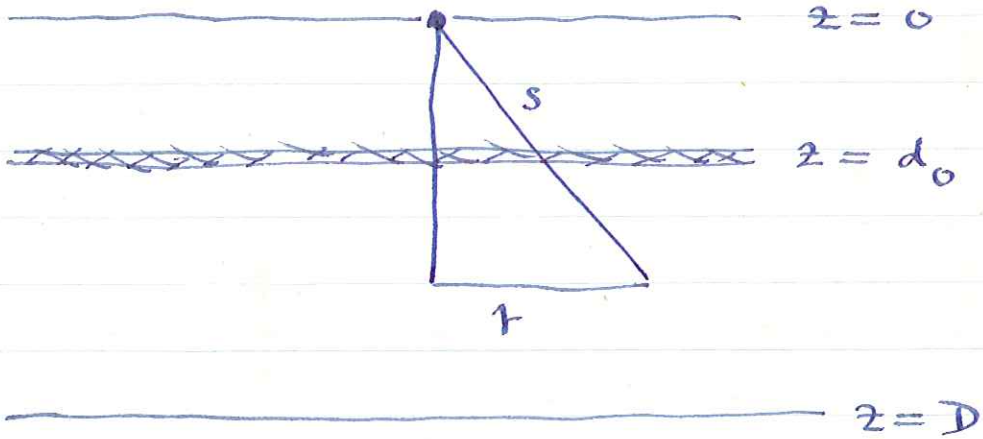
We integrate over rings of mass below the observation point:

call this δV

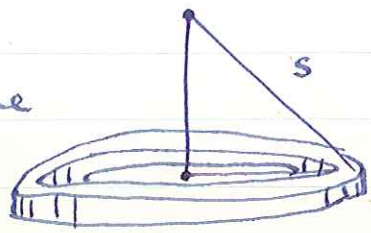
find V here (say)



We consider the model (cylindrical coords centered on obs. pt.)



Rings: all pts. same dist. s from.



Integrate out a distance L.

$s = \sqrt{r^2 + z^2}$, ϕ = azimuthal angle

Then

call this δV

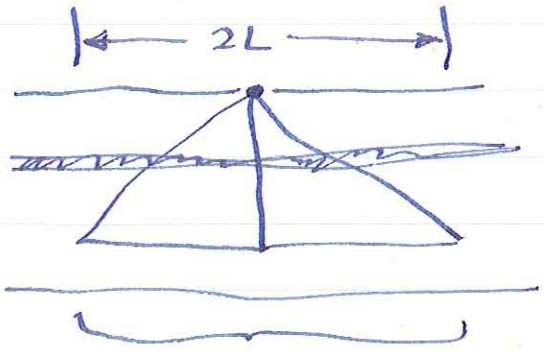
$$V = -G \int_0^{2\pi} \int_0^{L} \frac{(\rho_w - \rho_m)(d - d_0)}{\sqrt{r^2 + d_0^2}} \, r \, dr$$

$$- G \int_0^{2\pi} \int_{d_0}^{D} dz \int_0^{L} \frac{\delta\rho(x,z)}{\sqrt{r^2 + z^2}} \, r \, dr$$

Top integral :

$$- 2\pi G \int_0^{L} \frac{(\rho_w - \rho_m)(d - d_0)}{\sqrt{r^2 + d_0^2}} \, r \, dr$$

Since integral only out to $r = L$ = distance over which d, $\delta\rho$ change insignificantly, can take d = const and $\delta\rho$ = const



lithosphere looks the same in this region since $D \ll L$.

let $u^2 = r^2 + d_0^2$

$u\,du = r\,dr$

top integral $= -2\pi G(\rho_w - \rho_m)(d-d_0)\left[\sqrt{L^2+d_0^2} - d_0\right]$

to next order this is $L + \frac{d_0}{2L}$

$\approx -2\pi G(\rho_w - \rho_m)(d-d_0)\,L$ ← since $d_0 \ll L$

bottom integral $= -2\pi G \int_{d_0}^{D} \delta\rho\, dz$ ← make this ∞

$\int_0^L \frac{r\,dr}{\sqrt{r^2+z^2}}$ ← have already replaced by zero

can't just replace this by L as above since z can be much larger than d_0.

$= -2\pi G \int_{d_0}^{D} \delta\rho \left[\sqrt{L^2+z^2} - z\right] dz$ ← $D \to \infty$

← can replace by 0 as before (actually, we've already done so)

Now since $D \ll L$ we know that $z \ll L$ so

$\sqrt{L^2+z^2} \approx L + \frac{z^2}{2L} + \cdots$

bottom integral $\approx -2\pi G\, L \int_0^D \delta\rho\, dz$ $D \to \infty$ here too

$-2\pi G \left(\frac{1}{2L}\right) \int_0^D z^2\, \delta\rho\, dz$

$+ 2\pi G \int_0^D z\, \delta\rho\, dz$

Altogether we get for $\delta V(x)$:

$$\delta V = 2\pi G L \left[(\rho_m - \rho_w)(d - d_0) - \int_0^D \delta\rho \, dz \right] + 2\pi G \int_0^D z \, \delta\rho \, dz - \pi G \int_0^D \frac{z^2}{L} \delta\rho \, dz + 2\pi G (\rho_M - \rho_W)(d - d_0) \frac{d_0^2}{2L}$$

The term $\propto L$ cancels exactly by the isostatic condition.

Isostasy is local; we are now free to take L as large as we desire, since the possibly troublesome term ($\propto L$) cancels out.

The third term is of relative order D/L compared to the second and can be neglected. Thus we get, finally,

$$\delta V(x) = 2\pi G \int_0^D z \, \delta\rho(x, z) \, dz$$

(can now let this depend on x again)

What is the corresponding geoid height anomaly?

In the absence of cooling and subsidence the potential due to

ρ_w $z = 0$

ρ_m $z = d_0$

is, say, $V_0(z)$. On the ϕ $V_0(z) = V_0(0)$ = const., as ϕ is an equipotential.

With cooling and subsidence ϕ gets displaced away from $z = 0$ by an amount $h(x)$ but keeps the same potential $V_0(0) =$ const. Thus.

$$V_0(0) = V_0(h) + \delta V(h) = \text{const.}$$

δV ← anomalous or extra potential just calculated

Taylor series:

Picture:

$$V_0(0) = V_0(0) + h V'(0) + \cdots + \delta V(0) + \cdots$$

Neglect .... terms. Now note that $V'(0) \equiv (dV_0/dz)_{z=0} = -g_0$ = accel. of gravity at level $z = 0$ = 980 cm/s^2.

Thus the geoid elevation w.r.t. $z = 0$ (flat ≡ actually, the hydrostatic ellipsoid) is given by

$$h(x) = + \frac{1}{g_0} V(0,x) \Big|_{z=0}$$

— if $h(x)$ is measured positive downwards

or, finally,

$$h(x) = +(2\pi G/g_0) \underbrace{\int_0^D z\, \delta\rho(x,z)\, dz}$$

note: geoid signal at any point depends only on mass distr. below that point if DKL. lithospheric density

dipole moment of anomalous

Summarized by Turcotte & Schubert pp. 522-29

This relation between h and the local dipole moment of the [redacted] density anomaly, in the case of an isostatically compensated density distribution due to Turcotte + Ockendon JGR (1977).

We shall make use of it in a moment.

This often taken to be the definition of the lithosphere thickness — thermal boundary layer — whether 95 % or some other % is arbitrary.

How do we calculate the subsidence and the geoid anomaly.

Because of thermal contraction the cooled region becomes more dense than the ambient mantle density ρ_m.

The anomalous density with depth as a function of age is

$$\delta\rho(z,t) = \rho_m \alpha \left[T_m - T(z,t) \right]$$

where α is the (volumetric) thermal expansion coefficient.

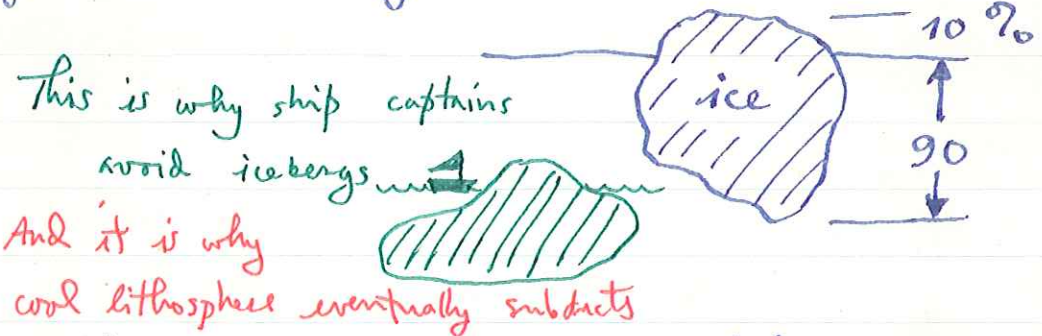
defn: α = fractional increase in volume $\delta V / V$ of a sample due to a 1°C. rise in temperature.

An increase in volume corresponds to a decrease in density $\delta\rho / \rho_m = - \delta V / V$

A typical value of α for igneous rocks is $\alpha \sim 3 \cdot 10^{-5} / °C$.

Cooling by 1000° C. thus increases the density by 3% from $\rho_m = 3.33$ g/cm^3 to 3.43 gm/cm^3. This is about how much <u>denser</u> the lithosphere is than the asthenosphere.

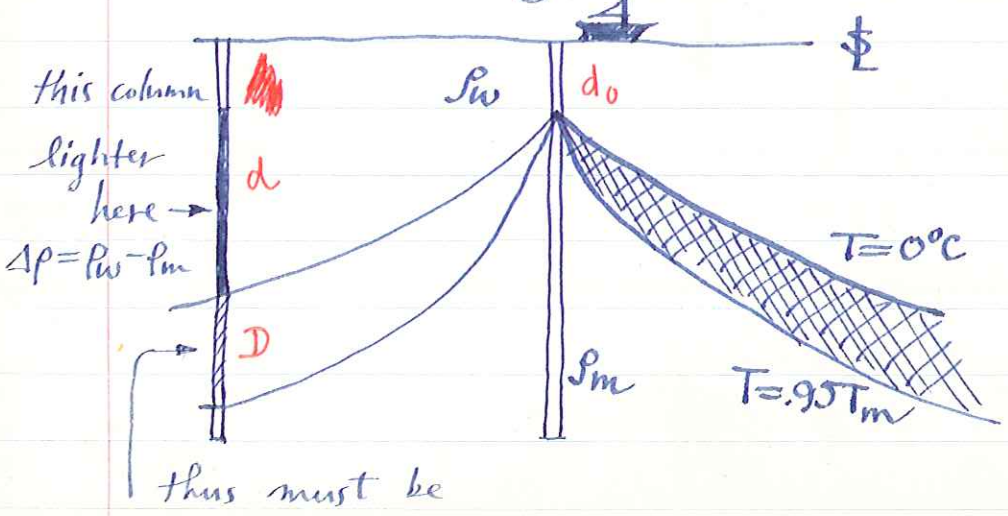
Note the <u>inherent instability</u> of the lithosphere. The isostatic balance is the opposite of that usually considered, e.g. an iceberg



This is why ship captains avoid icebergs

And it is why the cool lithosphere eventually subducts

But in case of <u>oceanic lithosphere</u>:



thus must be heavier here

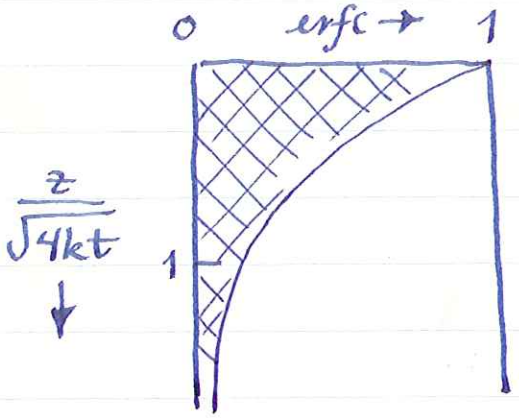
This unstable situation thought to be one of the main <u>driving forces</u> of plate tectonics.

$$\delta\rho = \rho_m \alpha T_m \underbrace{\left[1 - \text{erf}\left(z/\sqrt{4kt}\right)\right]}$$

this called erfc: complementary error function

$$\boxed{\delta\rho(z,t) = \rho_m \alpha T_m \, \text{erfc}\left(z/\sqrt{4kt}\right)}$$

Plot of erfc looks like



picture of anomalous density: greatest at top decreases with depth

By isostatically balancing columns we derived the formula for the subsidence

we now take D to be ∞

$$(\rho_m - \rho_w)(d - d_0) = \int_0^{\infty} \delta\rho(x,z)\, dz$$

Alternatively we could have taken the solution

$$T(z,t) = T_m \operatorname{erf}\left(\frac{z-d}{\sqrt{4kt}}\right)$$

This would satisfy upper b.c. at $z = d$ rather than $z = 0$. But these are niceties.

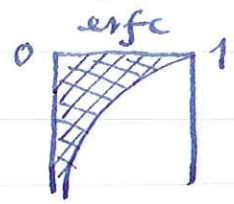
$$\int_0^\infty \delta\rho \, dz = \rho_m \alpha T_m \int_0^\infty \operatorname{erfc}\left(\frac{z}{\sqrt{4kt}}\right) dz$$

$$= \rho_m \alpha T_m (4kt)^{1/2} \int_0^\infty \operatorname{erfc} u \, du$$

$$u = \frac{z}{\sqrt{4kt}}$$

$$dz = \sqrt{4kt}\, du$$

The integral is just the shaded area.



Can be shown that

$$\int_0^\infty \operatorname{erfc} u \, du = \frac{1}{\sqrt{\pi}}$$

Thus we get

$$d = d_0 + 2\left(\frac{\rho_m}{\rho_m - \rho_w}\right)\alpha T_m \left(kt/\pi\right)^{1/2}$$

The subsidence of the seafloor should vary as $t^{1/2}$ where t is the age.

This is an incredibly simple prediction for such a complicated tectonic process as sea-floor spreading must be.

Essentially the $t^{1/2}$ dependence comes from fact that thickness of cooled layer goes like $t^{1/2}$ and subsidence depends on thermal contraction.

For the geoid signal, relative to the rise crest, we derived the formula

$$h = + \frac{2\pi G}{g} \underbrace{\int_0^\infty z\, \delta\rho\,(x,z)\, dz}$$

dipole moment of anomalous density.

$u = z/\sqrt{4kt}$

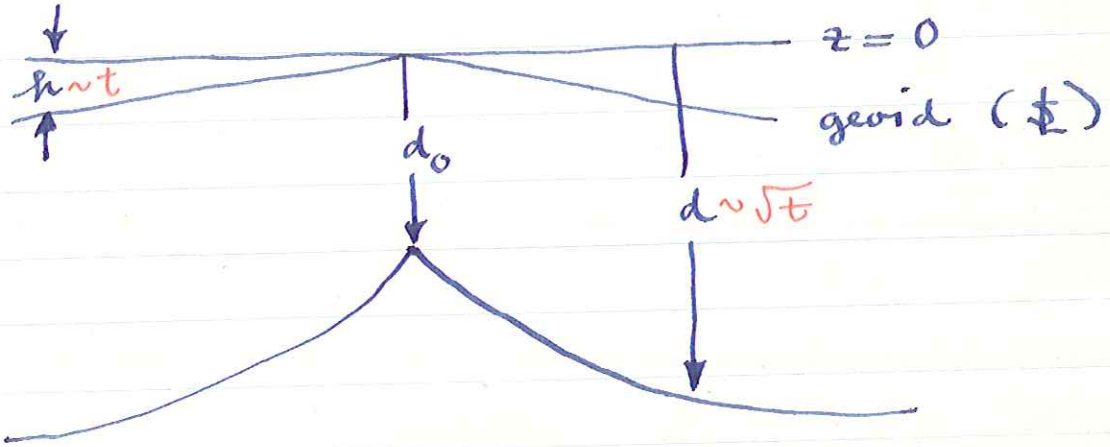
$$h = +(2\pi G/g) \int_0^\infty z \operatorname{erfc}(z/\sqrt{4kt})\, dz$$

$$= + \frac{2\pi G}{g} (4kt) \underbrace{\int_0^\infty u \operatorname{erfc} u \, du}_{\text{this definite integral has value } 1/4}$$

Thus

$$\boxed{h = + \frac{2\pi G}{g} \rho_m \alpha T_m k t}$$

The geoid anomaly is everywhere ~~negative~~ lower than at the ridge and $\propto$ age t.



Picture highly distorted since actually $h \ll d$.

The lab exercise writes

$$d - d_0 = c_1 \sqrt{t}$$
$$h = + c_2 t$$

where c_1 is as we have derived and

$$c_2 = \frac{2\pi G}{g} \rho_m \alpha (T_m - T_0) \kappa \left[1 + \frac{2\rho_m \alpha (T_m - T_0)}{(\rho_m - \rho_w)\pi} \right]$$

here T_0, the upper temp not necessarily zero

this factor extra has value 1.04

The difference comes from a slightly different point of view (actually due to integrating 2 times $\Delta\rho = \rho_w - \rho_m$ in region $d_0 < z < d$). Extra factor of 1.04 is significant compared to measurement error.

Using "typical" values (those adopted by Parsons and Sclater 1977)

$\alpha = 3.2 \cdot 10^{-5}$ °C^{-1}
$T_m = 1350$ °C.
$\kappa = 0.008$ cm^2/s
$\rho_m = 3.33$ g/cm^3
$\rho_w = 1.0$ g/cm^3
$g = 980$ cm/s^2

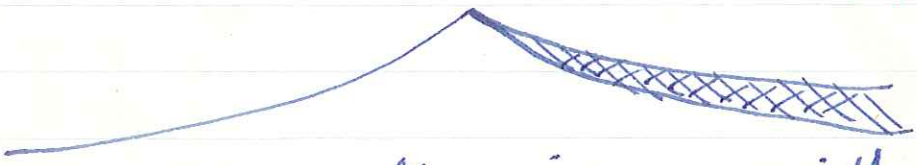
$$C_1 = 350 \text{ m} / (\text{m.y.})^{1/2}$$

$$C_2 = 0.15 \text{ m} / \text{m.y.}$$

How does this agree with observations? This is the object of the lab. exercise. Recent study of Heestand and Crough of depth vs. $(\text{age})^{1/2}$ tries to "correct" for hot spot bias by eliminating points near (within 1000 km) hotspots. Finds

$$d = 2500 \text{ m} + 320 \text{ m} / (\text{m.y.})^{1/2}$$

Also must correct for <u>sediment thickness</u>



(since they are not as dense as $\rho = 3.3$ lithosphere)

generally increases with age (more time for bugs to fall on top)

known from seismic profiling.

Haxby 1980 or 1981 study of GEOS-3 data from MAR + Reykjanes.

Here one must correct for broad regional geoid bumps (sources below lithosphere)

The h vs. t is seen very nicely and agrees very well with the prediction

$$h = 0.15 \text{ m} / (\text{m.y.})$$

15 m anomaly at 100 m.y.

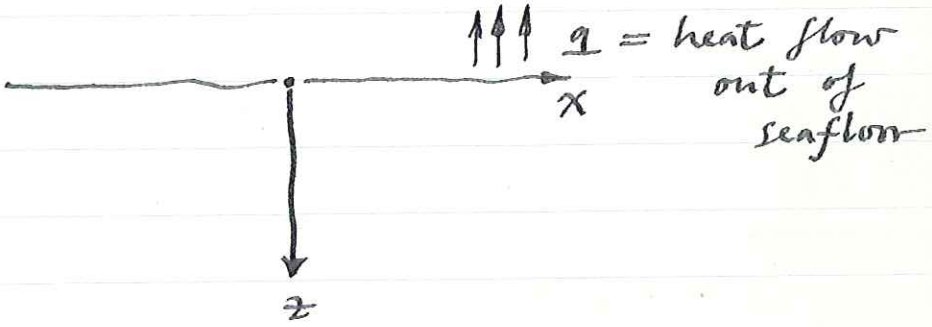
This excellent agreement is very strong evidence for the isostatic compensation of the lithosphere. Even a small deviation from isostasy would affect this result significantly, as you'll see from homework problem.

Currently a controversy over whether d vs. $t^{1/2}$ continues out to very old basins (200 m.y.). If all data is considered, a flattening of d vs. $\sqrt{t}$ is seen beyond 80 m.y. Very old basins do not continue to subside. No longer cools like a half-space. Would imply an extra source of heat from below.

This phenomenon first described quantitatively by Parsons and Sclater 1977 who found $c_1 = 350$ m/m.y$^{1/2}$ out to 80 m.y.

But when hot-spot contamination corrected for might be like $\sqrt{t}$ for [redacted] even oldest basins. Trouble: in N. Atlantic studied by Heestand + Crough ∃ no old ocean floor far from hot spots.

One more measured quantity can be predicted by our model: the heat flow.



heat flow $q = \kappa \left(dT/dz\right)_{z=0}$

$$T = \frac{2}{\sqrt{\pi}} T_m \int_0^{z/\sqrt{4kt}} e^{-u^2}\, du$$

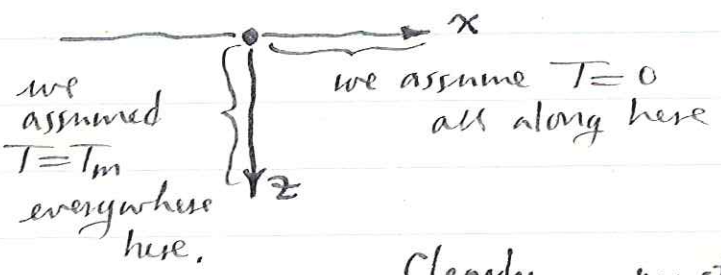
$$\frac{dT}{dz} = \frac{2}{\sqrt{\pi}} T_m \, e^{-z^2/4kt} \, (4kt)^{-1/2}$$

$$\left(dT/dz\right)_{z=0} = T_m \left(\pi k t\right)^{-1/2}$$

Thus we predict that, since $\kappa = \rho_m c_p k$,

$$\boxed{q = \rho_m c_p T_m (k/\pi t)^{1/2}}$$

The heat flow should vary like $t^{-1/2}$. The singularity at $t=0$ (the rise crest) shows that our solution breaks down very near the rise crest. Can be improved by modifying the b.c.



Clearly, must exhibit some kind of singularity. This not only due to our approx. that heat flows only vertically. Need to modify b.c. on vertical axis.

But this effects soln only very near ridge (within the axial valley where active volcanism is occurring).

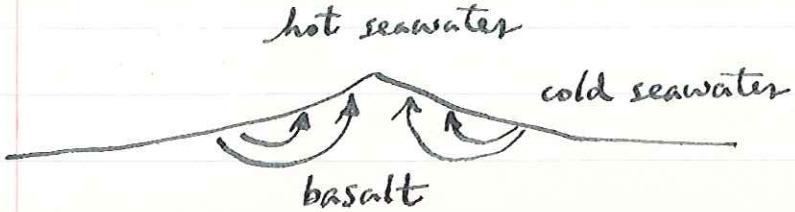
How does our solution compare with data elsewhere?

Recall the scatter and extreme local variability of measurements near the rise axis (ages < 30–40 m.y.).

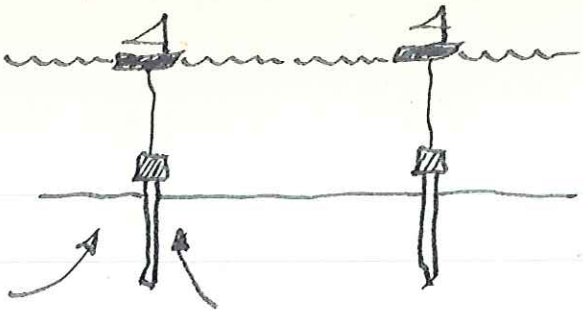
Scatter most extreme in poorly sedimented areas. Sediment cover increases with age.

If averages of <u>all measurements</u> taken and plotted vs. age agreement with q vs. $t^{-1/2}$ is pretty good for $t > 30–40$ m.y. but poor for $t < 30–40$ m.y. (see pts , , in Fig. 10 of Parsons + Sclater 1977). Theoretical <u>curve</u> uses values for ρ_m, c_p, k given previously.

Discrepancy for $t < 30–40$ m.y. and scatter now both attributed to <u>hydrothermal circulation</u> in basaltic oceanic crust.



This can account for local variability.



near hot spring: much of heat transported as hot H_2O, get a low measurement

not near hot spring: heat conduction dominates, get a reliable measurement

Only the <u>conducted</u> heat flow is measured.
The hot springs are thought to be most frequent in poorly sedimented areas.
Sediment cover seems to seal off cracks and vents in the basalt and influence the circulation.

If so-called "reliable" q averages are plotted (values in regions with uniform sed. cover > 200 m. thick) are plotted ⧫ the agreement is much better.

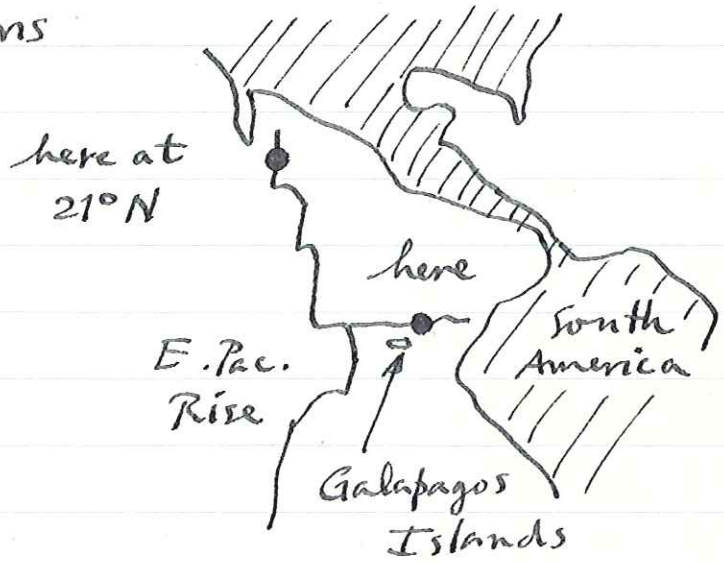
If take log:

$$\log q = \text{const} - \tfrac{1}{2} \log t$$

Slope of a log-log plot should be -1/2. The agreement, shown in Fig. 11 of P+S, is quite good from 5 to 150 m.y.

Existence of submarine hot springs first inferred on basis of heat flow measurements.

First direct evidence 1977 Alvin submersible found hydrothermal vents 2 locations



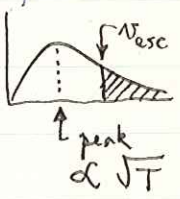
Hot springs on Galapagos Ridge 17°C.
" compared to 2°C. ambient bottom temp.
Black smokers" at 21°N however almost 400°C, kept from boiling by seabottom pressure. Galapagos vents diluted by cold seawater leakage from sides.

Interesting fauna, e.g. large red clots of hemoglobin because of low O_2 content) clams.

Can estimate amount of heat flowing out in form of hot H_2O by an argument based on Helium 3. Helium inert and very light, escapes from our atmosphere quickly, would not be in atmosphere unless constantly replenished. Most of this coming from oceanic hot springs.

one finds tail of Maxwell-Boltzmann distribution > escape velocity from ⊕

Can calculate rate of escape to space $\approx 3 \cdot 10^{19}$ atoms/sec. This much must be leaking out of ⊕ to keep atmospheric concentration in steady state.



this ratio same for both cold Galapagos and hot 21°N; ↓ evidence for leaky plumbing of former.

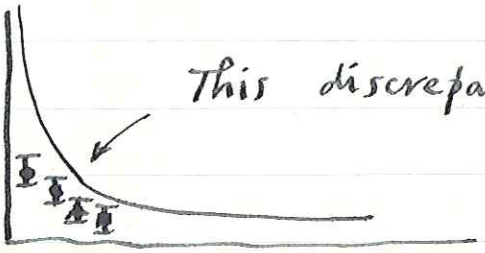
Can measure ratio of He atoms / calorie in water samples collected by Irwin at vents ($8 \cdot 10^{-8}$ cal / atom of He 3)

$$3 \cdot 10^{19} \frac{\text{atoms}}{\text{sec}} \times 3 \cdot 10^{7} \frac{\text{sec}}{\text{yr}} \times 8 \cdot 10^{-8} \frac{\text{cal}}{\text{atom}}$$

Estimated heat escaping from all oceanic hot springs $7 \cdot 10^{19}$ cal/yr
or about $10 \cdot 10^{12}$ W (J/sec)
10 Terrawatts

Can be compared with amount that "all data" averages fall below the theoretical heat flow

9



This discrepancy estimated by Langseth and Anderson to be $8 \cdot 10^{12}$ W in excellent (!) ~~reasonable~~ agreement with above estimate.

Importance of vents for oceanic chemistry is profound. From concentration of He 3 can estimate total flow of seawater through the crustal plumbing Get 6000 m^3/s, about 1/3 flow of Mississippi at New Orleans.

Entire ocean cycled through seafloor in 5–10 m.y. or about 500 times since world began. Many chemical exchanges occur during this cycling, solves many previously perplexing geochemical problems, e.g. known for years that conc. of Mg in oceans less than that flowing in in rivers. Reason: gets stripped out during cycling through crust.

Hot springs also a source of sulfide ore deposits. Iron, copper, other heavy metal sulfides utterly insoluble in H_2O at ordinary temps. but soluble in 400°C. The instant the water emerges and cools, sulfides precipitate out, "black smokers" are precipitating sulfides.

By viewing the films and videotapes of the "black smokers" one can estimate the total flow rate coming out of a single hydrothermal vent system.

At 21° N it is estimated to be $v = 1-5$ m/s and radius $r = 10-20$ cm. The total heat flow out the system is

$$q = \pi r^2 v \rho_w c_p T$$

(T: 350° – 400° C.)

$$\approx 6 \cdot 10^7 \text{ cal/sec}$$

The total hydrothermal heat flow, estimated from the ^3He / calories, is $5 \cdot 10^{19}$ cal/yr $\sim 1.7 \cdot 10^{12}$ cal/sec.

The estimated number of hydrothermal events required on all the world's ridge systems is thus about 25,000 – 30,000.

This is one for every 4–7 km of ridge length, which is why they're hard to find in the dark under 2.5 km of seawater.